1. $$\frac{dx}{dt} = 4x - 2xy$$
$$\frac{dy}{dt} = -3y + 3xy$$

(a) x: prey, y: predators

(b) $4x - 2xy = 2x(2-y) = 0 \longrightarrow \boxed{x=0 \downarrow y=0}$ or $\boxed{y=2,\ x=1}$

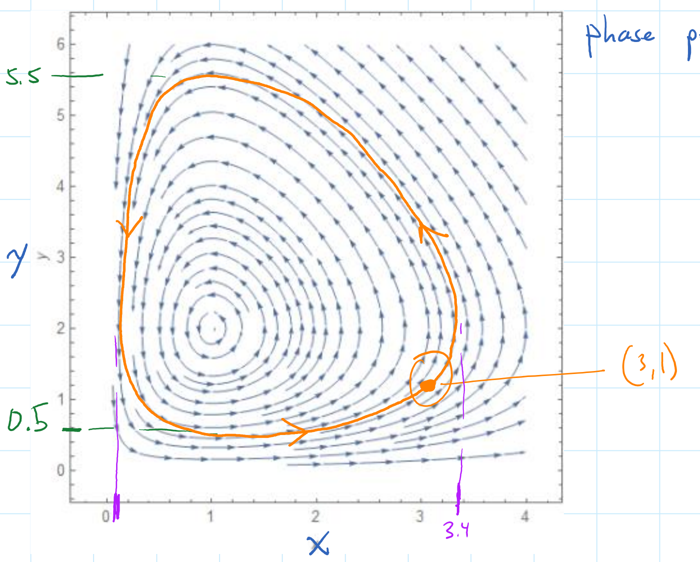
$-3y + 3xy = 3y(-1+x) = 0$

equilibrium solutions: $(0,0)$, $(1,2)$

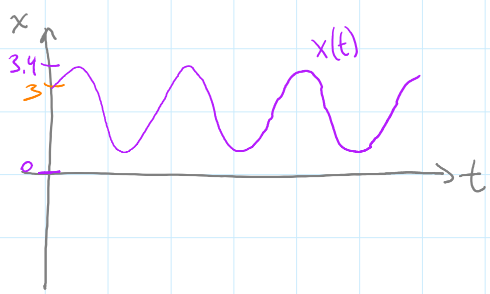
(c) If $x=3$, $y=1$, $\frac{dx}{dt} = 6$ so x pop. is incr,

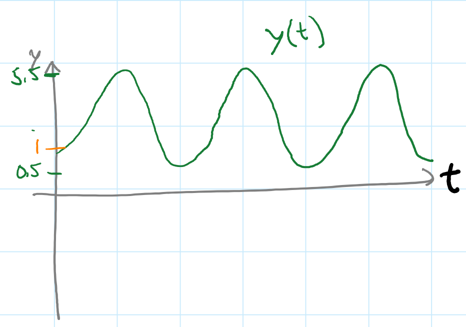
and $\frac{dy}{dt} = 6$ so y pop. is incr. at same rate

(d) phase portrait



(e)





2. $\frac{dx}{dt} = 2(1 - x)$ ...

2. $\frac{dx}{dt} = 2x\left(1 - \frac{x}{3}\right) - xy$

$\frac{dy}{dt} = 3y\left(1 - \frac{y}{5}\right) - 3xy$

(a) Competing: both xy terms have negative coefficients

(b) $\frac{dx}{dt} = 2x\left(1-\frac{x}{3}\right) - xy = x\left(2\left(1-\frac{x}{3}\right) - y\right) = x\left(2 - \frac{2}{3}x - y\right) = 0$

$\frac{dy}{dt} = 3y\left(1-\frac{y}{5}\right) - 3xy = 3y\left(1 - \frac{y}{5} - x\right) = 0$

eq. sols:
$(0,0), (3,0), (0,5)$
$\left(\frac{9}{13}, \frac{20}{13}\right)$

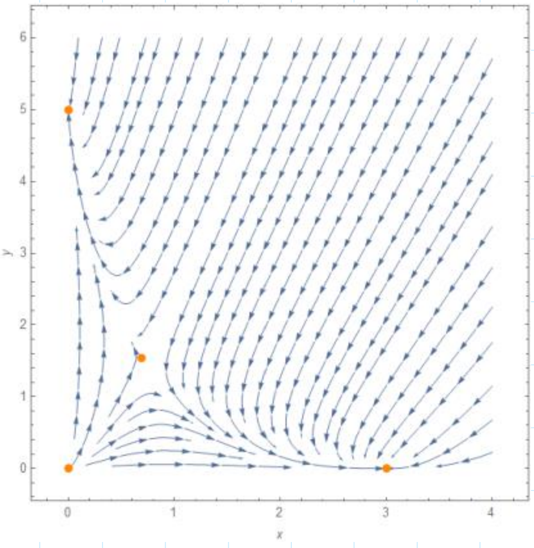
If $y=0$, then either $x=0$ or $2 - \frac{2}{3}x - 0 = 0$
$2 = \frac{2}{3}x$, so $x = 3$

If $1 - \frac{y}{5} - x = 0$, then either $x=0$ or $2 - \frac{2}{3}x - y = 0$

$1 - \frac{y}{5} - 0 = 0$
$y = 5$

$$\begin{cases} 1 - \frac{y}{5} - x = 0 \\ 2 - \frac{2}{3}x - y = 0 \end{cases}$$ solution: $\left(\frac{9}{13}, \frac{20}{13}\right)$

(c) If $x(0) = 4$ and $y(0) = 0$, the y population doesn't exist. The x-population decreases to its equilibrium point at $x = 3$.



(d) If $x(0) = 1$ and $y(0) = 1$, then $\frac{dx}{dt} > 0$ while $\frac{dy}{dt} < 0$. From the phase portrait, it looks like the system approaches the equilibrium point $(3,0)$.

(e) Other possible behaviors include approaching the equilibrium point $(0,5)$ or possibly approaching the equilibrium point $\left(\frac{9}{13}, \frac{20}{13}\right)$.