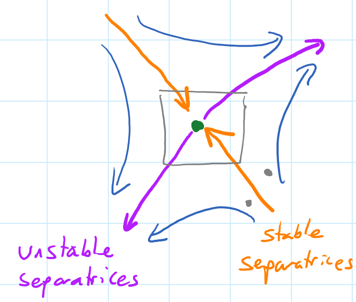
EXAM 2:
A: 60 and up
B: 50 to 59
C: 40 to 49

FROM §5.1 READING:

The solutions that go directly towards or away from a saddle point are called **SEPARATRICES** — they separate solutions with different long-term behavior.



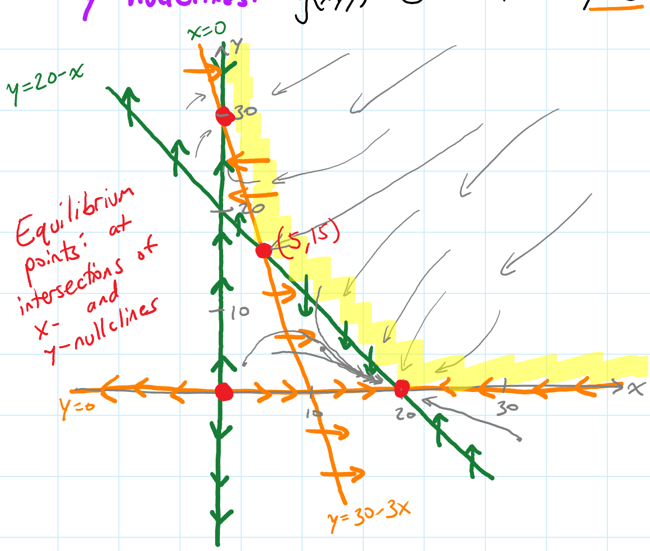
DEFINITION: For the system $\begin{cases} \frac{dx}{dt} = f(x,y) \\ \frac{dy}{dt} = g(x,y) \end{cases}$

the **x-NULLCLINE** is the set of points where $f(x,y)=0$ (solution curves have vertical tangents) and the **y-NULLCLINE** is the set of points where $g(x,y)=0$ (solution curves have horizontal tangents).

EXAMPLE:
$$\frac{dx}{dt} = x(20-x-y) = f(x,y)$$
$$\frac{dy}{dt} = y(30-3x-y) = g(x,y)$$

x-nullclines: $f(x,y)=0$ if $x=0$ or $y=20-x$ (vertical tangents)

y-nullclines: $g(x,y)=0$ if $y=0$ or $y=30-3x$ (horizontal tangents)



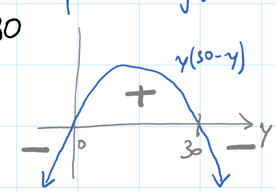
In what direction do solutions cross the nullclines?

- **x=0:** We know $\frac{dx}{dt}=0$

Consider $\frac{dy}{dt} = y(30-3x-y) = \underbrace{y(30-y)}_{\text{pos or neg?}}$

$\frac{dy}{dt}$ is + if $0<y<30$

$\frac{dy}{dt}$ is − if $y<0$ or $y>30$

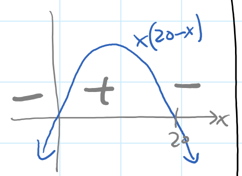


• $y=0$: We know $\frac{dy}{dt}=0$

Consider $\frac{dx}{dt} = x(20-x-y) = x(20-x)$

$\frac{dx}{dt}$ is $+$ if $0<x<20$

$\frac{dx}{dt}$ is $-$ if $x<0$ or $x>20$



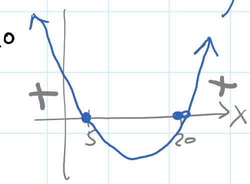
• $y = 20-x$ We know $\frac{dx}{dt}=0$

Consider $\frac{dy}{dt} = y(30-3x-y) = (20-x)(30-3x-(20-x))$

$= (20-x)(10-2x) = (20-x)(5-x)(2)$

$\frac{dy}{dt}$ is $+$ if $x<5$ or $x>20$

$\frac{dy}{dt}$ is $-$ if $5<x<20$



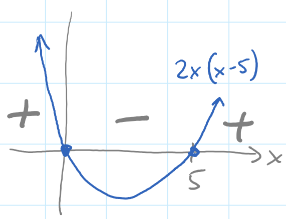
• $y = 30-3x$ We know $\frac{dy}{dt}=0$

Consider $\frac{dx}{dt} = x(20-x-y)$

$= x(20-x-(30-3x))$

$= x(-10+2x) = 2x(x-5)$

$2x^2-10x$



$\left.\begin{array}{l}\frac{dx}{dt} \text{ is } - \text{ if } 0<x<5\end{array}\right]$ go left

$\left.\begin{array}{l}\frac{dx}{dt} \text{ is } + \text{ if } x<0 \\ \quad \text{or } x>5\end{array}\right]$ go right

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EXAMPLE:

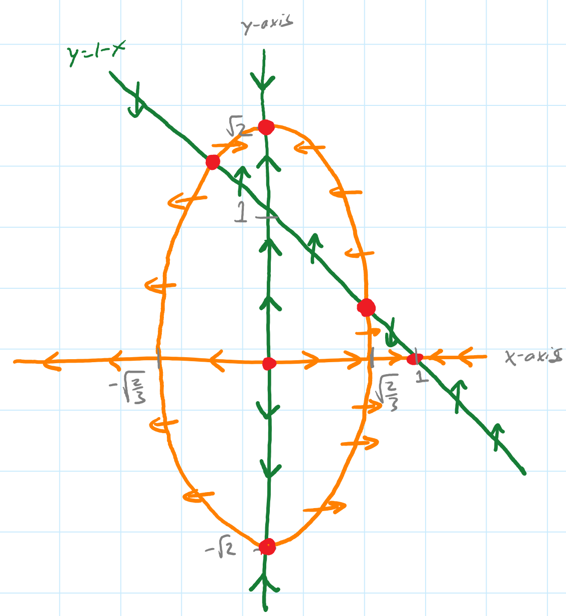
$$\frac{dx}{dt} = x - x^2 - xy = \underline{x}\,\underline{(1-x-y)}$$

$$\frac{dy}{dt} = 2y - y^3 - 3x^2y = \underline{y}\,\underline{(2-y^2-3x^2)}$$

x-nullclines: $\underline{x=0}$ or $\underline{y=1-x}$

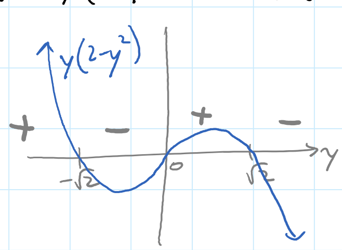
y-nullclines: $\underline{y=0}$ or $\underline{2=3x^2+y^2}$

ellipse



• $x=0$: We know $\frac{dx}{dt}=0$.

Consider: $\frac{dy}{dt} = y(2-y^2-3x^2) = y(2-y^2)$



• $y=1-x$: We know $\frac{dx}{dt}=0$. Consider $\frac{dy}{dt} = y(2-y^2-3x^2) = (1-x)(2-(1-x)^2-3x^2)$

$$= (1-x)(1+2x-4x^2)$$

$$x=1 \qquad x = \frac{-2 \pm \sqrt{4-4(-4)}}{2} = -1 \pm \sqrt{5}$$

Along $y=1-x$, $\frac{dy}{dt}$ changes from $-$ to $+$ to $-$ to $+$ at 3 equilibrium points

• $y=0$: We know $\frac{dy}{dt}=0$. Consider $\frac{dx}{dt} = x-x^2-xy = x-x^2 = x(1-x)$

$\frac{dx}{dt}<0$ if $x<0$ or $x>1$; $\frac{dx}{dt}>0$ if $0<x<1$.

• $2=3x^2+y^2$: We know $\frac{dy}{dt}=0$. Consider $\frac{dx}{dt} = x(1-x-y)$

This is tricky! Observe that: $\frac{dx}{dt}>0$ if x and $1-x-y$ have the same sign.

$\frac{dx}{dt}<0$ if x and $1-x-y$ have opposite signs.

Thus, the arrows shown on the ellipse can be determined.