

## SIMULATION OF RANDOM EXPERIMENTS

Basic commands for generating random numbers in R:

`runif(1)` — generates a random number between 0 and 1

`runif(3)` — generates 3 independent random numbers between 0 and 1

1. Simulate 10,000 flips of two unfair coins, one of which lands heads with probability 0.4, and the other lands heads with probability 0.6. Count the number of times that both land heads.

```
count <- 0          # count starts at zero
for(i in 1:10000){  # loop 10000 times
  # generate random numbers between 0 and 1
  r <- runif(1)
  s <- runif(1)

  # if both coins are heads, then increment counter
  if(r < 0.4 && s < 0.6){
    count <- count + 1
  }
}
print(count) # this is the number of times both coins land heads
```

→ count should be about 2400

```
# more concise code to solve the same problem as above
r <- runif(10000) # 10000 flips of the first coin
s <- runif(10000) # 10000 flips of the second coin
count = (r < 0.4) & (s < 0.6)
#print(count)
print(sum(count))
```

## SAMPLING IN R

Basic command:

`sample(n, size, replace)`

number of random elements that you want  
↓  
number or set to sample from  
↑  
TRUE or FALSE

EXAMPLES:  $\text{sample}(1:6, 5, \text{replace} = \text{TRUE})$

simulates 5 rolls of a standard, fair die

$\text{sample}(1:100, 20, \text{FALSE})$

choose 20 numbers from  $\{1, 2, \dots, 100\}$ , without replacement

2. Use simulation to approximate the probability that at least two 1s appear in three rolls of standard, fair dice.

```
# simulate the probability that at least two 1s appears in
# three rolls of a standard, fair die
c <- 0
for(i in 1:10000){
  dice <- sample(1:6, 3, TRUE) # three die rolls
  ones <- sum(dice == 1)       # number of ones in the rolls
  if(ones >= 2){
    c <- c + 1 # increment counter
  }
}
print(c/10000)
```

Analytic solution:

$$\begin{aligned} P(\text{at least two 1s}) &= P(\text{exactly two 1s}) + P(\text{exactly three 1s}) \\ &= \binom{3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^3 = \frac{16}{216} \approx 0.074 \end{aligned}$$