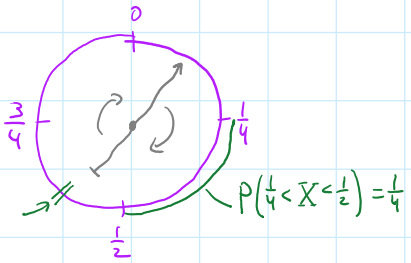
1. Give an example of a continuous random variable.

- rainfall in a month
- height of a building
- the value that a spinner stops at
- runif( )



DEF: A r.v. X is continuous if both of the following hold:

(1) The possible values of X form an interval or union of intervals

(2) $P(X = c) = 0$ for any $c \in \mathbb{R}$

PROBABILITY DENSITY FUNCTION (pdf): of a continuous rv X is a function $f(x)$ such that for $a \le b$, $P(a \le X \le b) = \int_a^b f(x)\,dx$.

NOTE: $P(X = a) = P(a \le X \le a) = \int_a^a f(x)\,dx = 0$

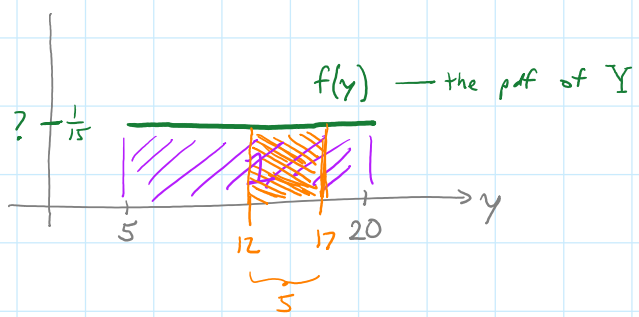
Also: $f(x) \ge 0$ for all x, and $\int_{-\infty}^{\infty} f(x)\,dx = 1$

2. Suppose that a continuous random variable Y has a pdf that is constant on the interval $[5, 20]$ and zero otherwise.

(a) What is the constant value of the pdf?

$(\text{Height})(15) = 1$

so height $= \frac{1}{15}$



(b) What is $P(12 < Y < 17)$?

$P(12 < Y < 17) = \int_{12}^{17} f(y)\,dy = \frac{1}{15}(5) = \frac{1}{3}$

$= \int_{12}^{17} \frac{1}{15}\,dy = \frac{y}{15}\Big|_{12}^{17} = \frac{1}{3}$

same as

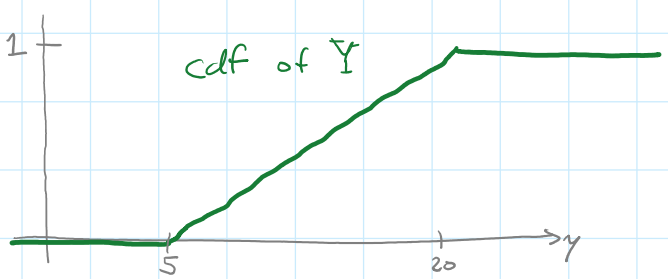
$P(12 \le Y \le 17)$

since $P(12 = Y) = 0$, $P(17 = Y) = 0$

(c) What is the cdf of Y?

$F(y) = P(Y \le y) = \int_5^y \frac{1}{15}\,dt$

$= \frac{t}{15}\Big|_5^y = \frac{y}{15} - \frac{5}{15}$



CUMULATIVE DISTRIBUTION FUNCTION (cdf) of a continuous rv X is defined $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$

pdf of X

PROPERTIES: $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to \infty} F(x) = 1$

$F(x)$ is non-decreasing (i.e., it's always increasing or constant)

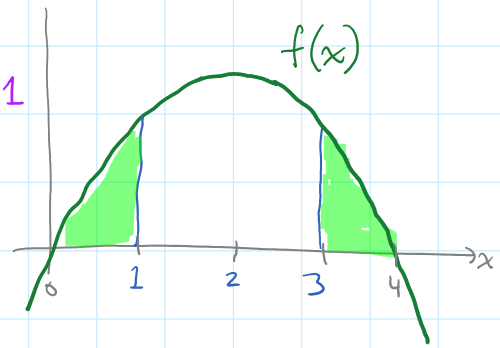
$F'(x) = f(x)$ ← Fundamental Theorem of Calculus

3. Suppose that a continuous random variable X has probability density function (pdf) given by $f(x) = kx(4-x)$ for $0 \le x \le 4$, and $f(x) = 0$ otherwise.

(a) What is the value of k?

$$\int_0^4 kx(4-x)\,dx = k\left(2x^2 - \tfrac{1}{3}x^3\right)\Big|_0^4 = \tfrac{32}{3}k = 1$$

so $k = \frac{3}{32}$



(b) Find $P(X > 3 \text{ or } X < 1)$.

$$P(X < 1) = \int_0^1 \tfrac{3}{32}x(4-x)\,dx = \tfrac{5}{32}$$

By symmetry, $P(X<1) + P(X>3) = 2 \cdot \frac{5}{32} = \frac{5}{16}$

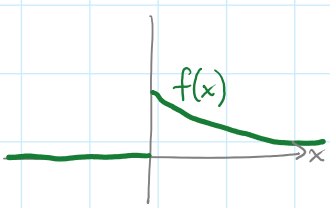
4. Suppose that the cdf of a random variable X is $F(x) = 1 - e^{-5x}$ for $x > 0$, and $F(x) = 0$ otherwise.

(a) What is the pdf of X?

differentiate $F(x)$:

$$f(x) = \frac{d}{dx}\left(1 - e^{-5x}\right) = 5e^{-5x} \quad \text{for } x > 0$$

$$f(x) = 0 \quad \text{for } x \le 0$$



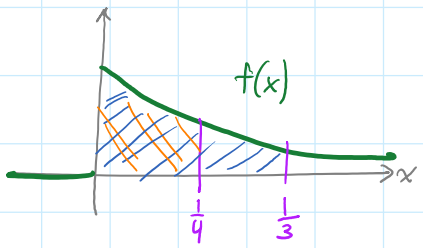
(b) What is $P\left(\frac{1}{4} < X < \frac{1}{3}\right)$? Can you get this from *either* the cdf or the pdf?

pdf: $P\left(\frac{1}{4} < X < \frac{1}{3}\right) = \int_{\frac{1}{4}}^{\frac{1}{3}} f(x)\,dx = \int_{\frac{1}{4}}^{\frac{1}{3}} 5e^{-5x}\,dx = -e^{-5x}\Big|_{\frac{1}{4}}^{\frac{1}{3}} = -e^{-5/3} + e^{-5/4} \approx 0.098$

$P\left(\frac{1}{4} < X < \frac{1}{3}\right) = P(X < \frac{1}{3}) - P(X < \frac{1}{4}) = F\left(\frac{1}{3}\right) - F\left(\frac{1}{4}\right)$

cdf: $P\left(\frac{1}{4} < X < \frac{1}{3}\right) = P\left(X < \frac{1}{3}\right) - P\left(X < \frac{1}{4}\right) = F\left(\frac{1}{3}\right) - F\left(\frac{1}{4}\right)$

$= \left(1 - e^{-5/3}\right) - \left(1 - e^{-5/4}\right)$



5. Let Y be a random variable with pdf given by $f(x) = \begin{cases} \frac{y}{2} & \text{if } 0 \le y \le 2, \\ 0 & \text{otherwise.} \end{cases}$

to be continued...

(a) Find a value $\eta_{0.25}$ such that $P(Y \le \eta_{0.25}) = 0.25$.

(b) What is the median of Y?