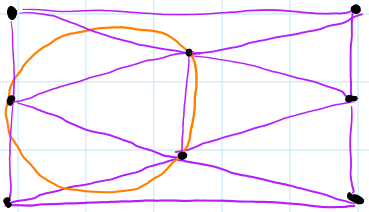
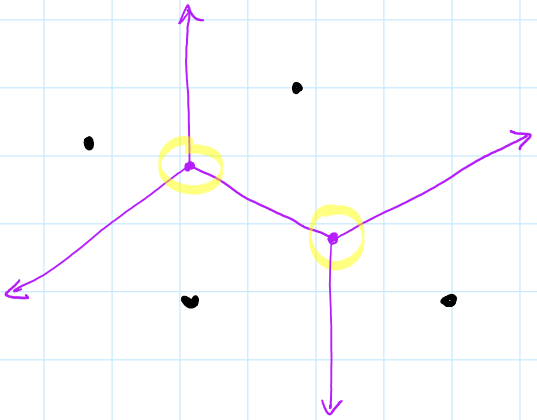
example:



QUESTION: If I have n sites, what is the max number of Voronoi vertices? ... of Voronoi edges?



$n = 4$

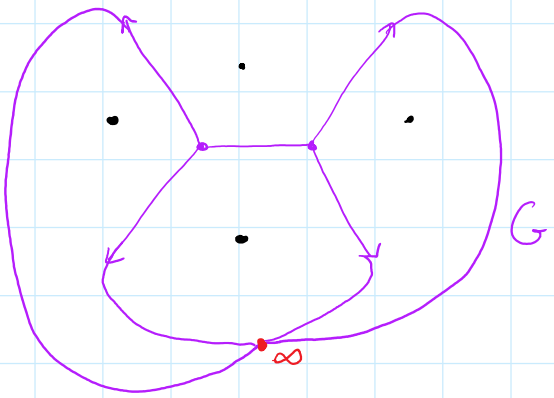
Voronoi vertices: 2

Voronoi edges: 5

MAX. NUMBERS OF VORONOI VERTICES AND EDGES

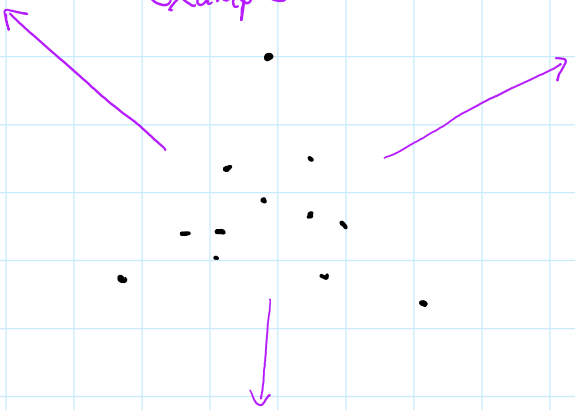
Let S be a point set with $n \geq 3$ sites, not all on the same circle. Then $Vor(S)$ has at most $2n-5$ vertices and $3n-6$ edges.

proof:



graph G has n faces (one per site)

Example



If Var(S) had e edges, then G also has e edges.

If Var(S) had v vertices, then G has $v+1$ vertices.

Euler's formula $V-E+F=2$: $(v+1)-e+n=2$

Add up the degrees of vertices in G: $2e \geq 3(v+1)$

Substitute: $v+1=2+e-n$

$$2e \geq 3(2+e-n)$$

$$2e \geq 6+3e-3n$$

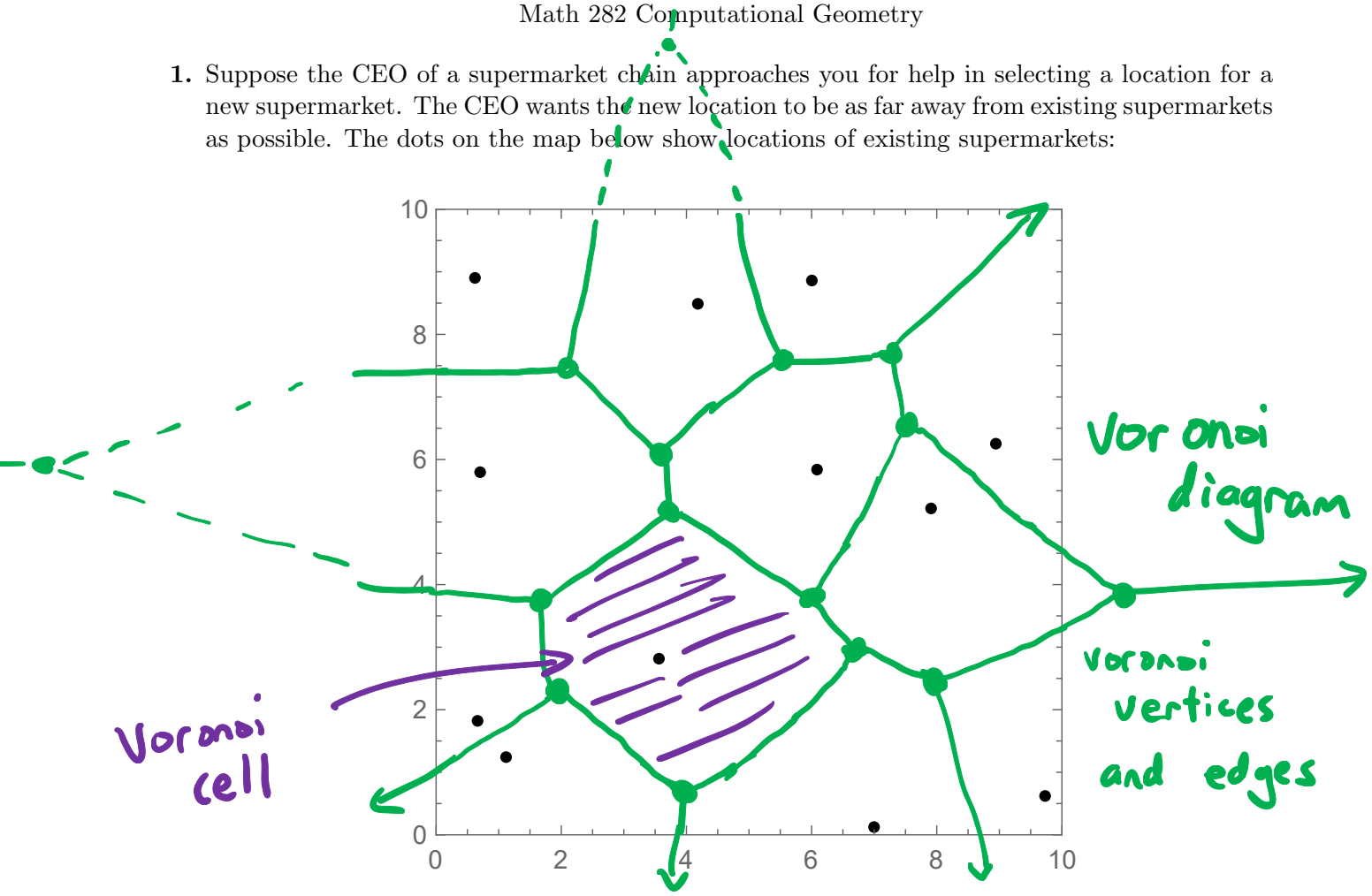
$$3n-6 \geq e$$

Substitute: $e=(v+1)+n-2 \Rightarrow 2n-5 \geq v$

Voronoi Diagrams

Math 282 Computational Geometry

1. Suppose the CEO of a supermarket chain approaches you for help in selecting a location for a new supermarket. The CEO wants the new location to be as far away from existing supermarkets as possible. The dots on the map below show locations of existing supermarkets:



Where would you position the new supermarket? Defend your choice.

2. Draw the Voronoi diagram whose "sites" are the existing supermarkets. How does the diagram change when you add a site for the new supermarket?

3. Fix $n \geq 3$. Find a point set S with n sites such that $\mathrm{Vor}(S)$ has the maximum possible number of vertices. How about the maximum possible number of edges?