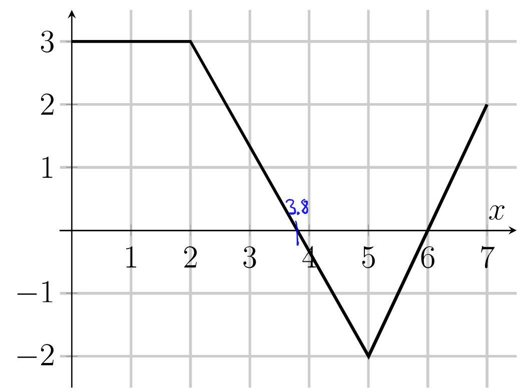
# The FTC and Indefinite Integrals

1. The graph of a derivative $f'$ is given below.




   (a) On what intervals is $f$ increasing? Decreasing?

   $f$ increasing where $f' > 0$: on the intervals $[0, 3.8)$ and $(6, 7]$

   $f$ decreasing where $f' < 0$: on the interval $(3.8, 6)$

   (b) On what intervals is $f$ concave up? Concave down?

   $f$ concave up where $f'$ increasing: on the interval $(5, 7)$

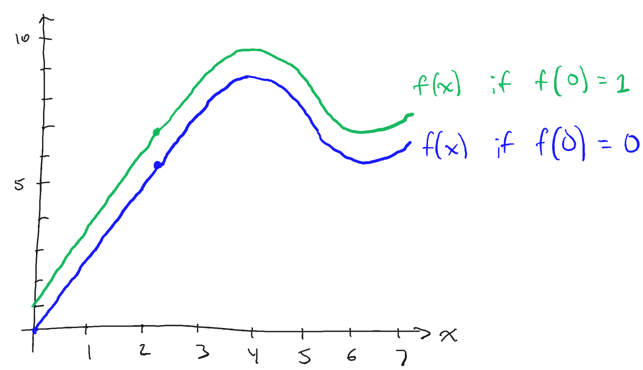
   $f$ concave down where $f'$ decreasing: on the interval $(2, 5)$

   (c) At what values of $x$ does $f$ have a local maximum? A local minimum? An inflection point?

   $f$ has a local max at $x = 3.8$

   $f$ has a local min at $x = 6$

   (d) Sketch a graph of $f$, if $f(0) = 0$. Then sketch the graph of $f$ if $f(0) = 1$.




2. How is $\int_a^b f(x)\,dx$ different from $\int f(x)\,dx$?

$\int_a^b f(x)\,dx$ is a number representing the signed area under the graph of $f(x)$

$\int f(x)\,dx$ is a family of functions: antiderivatives of $f(x)$

3. Evaluate the indefinite integrals.

(a) $\displaystyle\int \left(x + \sqrt{x}\right)\, dx$ $= \int (x + x^{1/2})\, dx = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C$

(b) $\displaystyle\int (5 + \sin(x))\, dx$ $= 5x - \cos x + C$

(c) $\displaystyle\int \left(e^{4+x} + e^{4x}\right)\, dx$ $= e^{4+x} + \frac{1}{4}e^{4x} + C$ *Hint:* $\frac{d}{dx}e^{4x} = ?$

(d) $\displaystyle\int \frac{1}{2x-1} dx$ $= \frac{1}{2}\ln|2x-1| + C$ *Hint:* $\frac{d}{dx}\left(\ln(2x-1)\right) = ?$

4. Let $F(x) = \displaystyle\int_0^x \left(t^2 - 10\right) dt$.

(a) Write a formula for $F(x)$ that does not involve an integral.

$$F(x) = \left[\frac{1}{3}t^3 - 10t\right]_{t=0}^{t=x} = \frac{1}{3}x^3 - 10x$$

(b) What is $F'(x)$?

$F'(x) = x^2 - 10$ ← note similarity to the integrand $t^2 - 10$

5. If $F(x) = \int_0^{x^2} (t^2 - 10)\, dt$, what is $F'(x)$?

$F(x) = \left[\frac{1}{3}t^3 - 10t\right]_{t=0}^{t=x^2} = \frac{1}{3}(x^2)^3 - 10(x^2) = \frac{1}{3}x^6 - 10x^2$

$F'(x) = 2x^5 - 20x$

FTC 2:

$F'(x) = ((x^2)^2 - 10)(2x)$

$= 2x^5 - 20x$

6. If $g(x) = \int_0^{x} \frac{\sin(t)}{t}\, dt$, what is $g'(x)$?

Use FTC 2: $g'(x) = \frac{\sin(x)}{x}$

7. If $F(x) = \int_1^{x^3} (s^6 + e^{4s})\, ds$, what is $F'(x)$?

By FTC2: $F'(x) = \left((x^3)^6 + e^{4x^3}\right)(3x^2)$

$= 3x^{20} + 3x^2 e^{4x^3}$

8. If $h(x) = \int_{3x}^{0} \cos(y)\, dy$, what is $h'(x)$?

First, $h(x) = -\int_0^{3x} \cos(y)\, dy$.

Then by FTC 2: $h'(x) = -3\cos(3x)$.

9. If $q(x) = \int_{\ln(x)}^{2x} (1 + t^2)\, dt$, what is $q'(x)$?

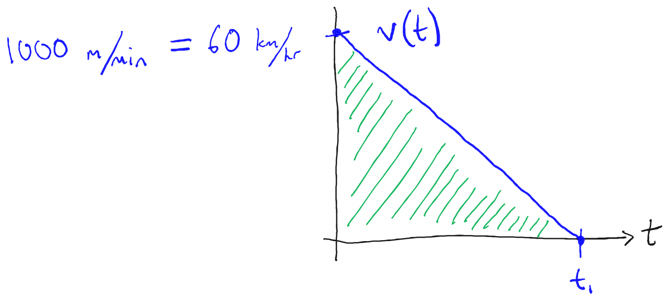
First: $q(x) = \int_0^{2x} (1+t^2)\, dt - \int_0^{\ln(x)} (1+t^2)\, dt$

Then by FTC 2: $q'(x) = 2(1 + (2x)^2) - (1 + (\ln(x))^2)\frac{1}{x}$

$q'(x) = 2 + 8x^2 - \frac{1}{x} - \frac{1}{x}(\ln x)^2$

10. A car, initially moving at 60 kilometers per hour, has a constant deceleration and stops in 40 meters. What is the deceleration of the car? In other words, the acceleration is $a(t) = c$ for what negative constant $c$?

Since acceleration is constant, the velocity graph has constant slope and looks like this:




The distance traveled is the shaded area: this is 40 meters.

Use the area of the triangle to solve for $t_1$:

$$40\,m = \frac{1}{2}(t_1)\left(1000\ m/min\right)$$ ← be consistent with units!

$$\frac{2(40\ m)}{1000\ m/min} = t_1 \qquad \text{or} \qquad t_1 = \frac{80}{1000}\ min = \frac{2}{25}\ min$$

The slope of $v(t)$ is then

$$\frac{1000\ m/min}{\frac{2}{25}\ min} = 12500\ m/min^2 = \frac{125}{36}\ m/sec^2 = 3.472\ m/sec^2$$

and this is the deceleration of the car.