Derivative Review

Work with the people at your table to see how much you can remember about derivatives!

1. The **derivative** of a function at a point represents the ______________ of the tangent line at that point. That is:

$$f'(a) = \text{“the } \underline{\hspace{3cm}} \text{ of the tangent line to the graph of } f(x) \text{ at } x = a\text{”}$$

2. Because of #1, the derivative can be used to understand the graph of a function $f(x)$:
   (a) If $f'(x) > 0$, then the graph of $f(x)$ is ______________.
   (b) If $f'(x) < 0$, then the graph of $f(x)$ is ______________.
   (c) If $f'(x) = 0$, then the graph of $f(x)$ is ______________.

3. What happens at points where the derivative changes sign?
   (a) If f' changes from positive to negative at $x = c$, then f has a local ______________ at $x = c$.
   (b) If f' changes from negative to positive at $x = c$, then f has a local ______________ at $x = c$.

4. The **second derivative** of a function at a point represents the ______________ of the graph at that point. That is:

$$f''(a) = \text{“the } \underline{\hspace{3cm}} \text{ of the graph of } f(x) \text{ at } x = a\text{”}$$

5. Because of #4, the second derivative can be used to understand the graph of a function $f(x)$:
   (a) If $f''(x) > 0$, then the graph of $f(x)$ is ______________.
   (b) If $f''(x) < 0$, then the graph of $f(x)$ is ______________

6. A point at which the graph of $f(x)$ changes concavity is called an ____________________.
   ⚠ **CAUTION:** Not every point at which $f''(x) = 0$ is one of these points!

7. Derivatives of specific kinds of functions:
   (a) **Power Rule:**
   If n is a real number and $f(x) = x^n$, then $f'(x) =$ ____________________.

   (b) **Trigonometric Functions**:
   If $f(x) = \sin(x)$, then $f'(x) =$ ____________________.

   If $f(x) = \cos(x)$, then $f'(x) =$ ____________________.

   If $f(x) = \tan(x)$, then $f'(x) =$ ____________________.

   (c) **Exponential Functions:**
   If $a > 0$ and $f(x) = a^x$, then $f'(x) =$ ____________________.

   In particular, if $f(x) = e^x$, then $f'(x) =$ ____________________.

   (d) **Natural Logarithm:**
   If $f(x) = \ln(x)$, then $f'(x) =$ ____________________.

8. Derivative “rules” for combining functions. You may assume that $f'(x)$ and $g'(x)$ both exist.

   (a) **Constant Multiple Rule:**
   If C is a constant, then $[C \cdot f(x)]' =$ ______________________.

   (b) **Function Sum/Difference Rule**:

   $$[f(x) + g(x)]' = \underline{\hspace{4cm}}.$$

   $$[f(x) - g(x)]' = \underline{\hspace{4cm}}.$$

   (c) **Product Rule:**

   $$[f(x)g(x)]' = \underline{\hspace{4cm}}.$$

   (d) **Chain Rule:** (composition of functions)

   $$[f(g(x))]' = \underline{\hspace{4cm}}.$$

   (e) **Quotient Rule:**

   $$\left[\frac{f(x)}{g(x)}\right]' = \underline{\hspace{4cm}}.$$

9. Let’s take some derivatives! Find f' and write down which method(s) you use for each:

   (a) $f(x) = x + \sqrt{x}$

   (b) $f(x) = 2 + \frac{1}{x} + \frac{1}{x^2}$

   (c) $f(x) = \sqrt{x}\cos(x)$

   (d) $f(x) = \left[x^2 + \sin(x)\right]^4$

   (e) $f(t) = e^{5+\ln t}$

   (f) $f(x) = 3\sqrt{x}\cos(x^2) + \pi$

10. Suppose that $g(x) = x^2 - 3x + 2$ and $f(x)$ is a differentiable function. All you know about $f(x)$ is the following:

$$\begin{array}{ll} f(0) = 3 & f'(0) = -1 \\ f(1) = 5 & f'(1) = 0 \\ f(2) = -2 & f'(2) = 3 \\ f(3) = 6 & f'(3) = 1 \end{array}$$

If possible:

(a) Find the derivative of $f(x)g(x)$ at $x = 1$.

(b) Find the derivative of $\frac{f(x)}{g(x)}$ at $x = 0$.

(c) Find the derivative of $f(g(x))$ at $x = 0$.

(d) Find the derivative of $\frac{f(x)+g(x)}{g(x)}$ at $x = 0$.

11. Let $f(x) = 2x^3 + 3x^2 - 36x$.

(a) On which intervals is f increasing? Decreasing?

(b) For which values of x does f have a local minimum? Local maximum?

(c) On which intervals is f concave down? Concave up? Where are the inflection points?

(d) Use this information to sketch a graph of f.

12. Find the maximum and minimum values of $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$.