

Calculus II – Day 2

Warm-up

1. Chloe was driving at a *constant* rate of 30 miles per hour for 2 hours.

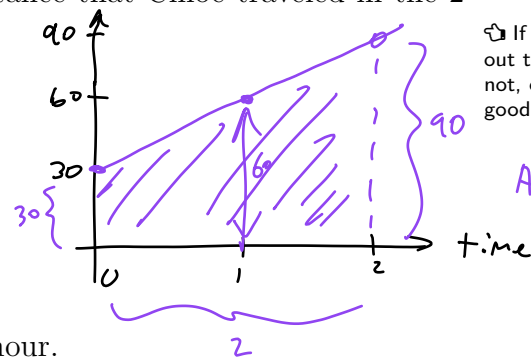
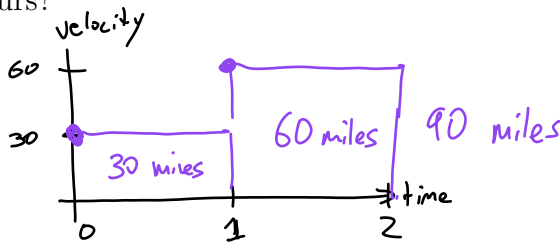
Group chat: How far did she go in the 2 hours? Make sure you can all explain exactly why the answer is what it is.

60 miles

What is 'group chat'? Discuss the question and what you think the answer is. If anyone has a different idea or answer (whether right or wrong), discuss that respectfully! Everyone will learn something and come out of it more knowledgeable.

2. Suppose you know that Chloe started driving at 30 miles per hour, but at the end of the first hour, she was traveling 60 miles per hour.

Group chat: Can you figure out *exactly* the distance that Chloe traveled in the 2 hours?



If yes, please figure out the answer! If not, can you give a good estimate?

3. Now Chloe decided to drive for 8 hours!

At the beginning, she was traveling 30 miles per hour.

At the end of 2 hours, she was traveling 60 miles per hour.

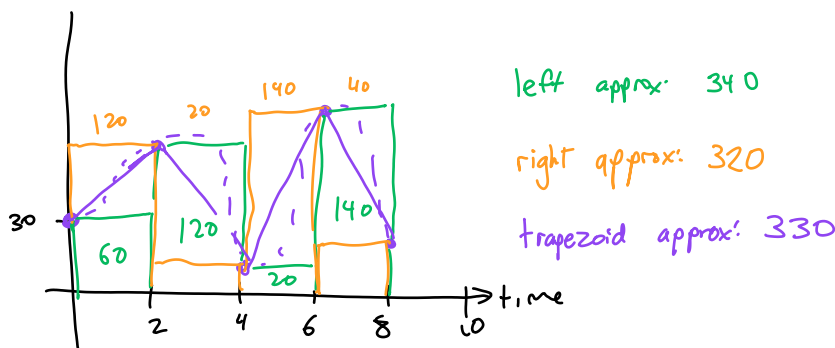
At the end of 4 hours, she was traveling 10 miles per hour.

At the end of 6 hours, she was traveling 70 miles per hour.

At the end of 8 hours, she was traveling 20 miles per hour.

Group chat: Can you figure out *exactly* the distance she traveled in the 8 hours?

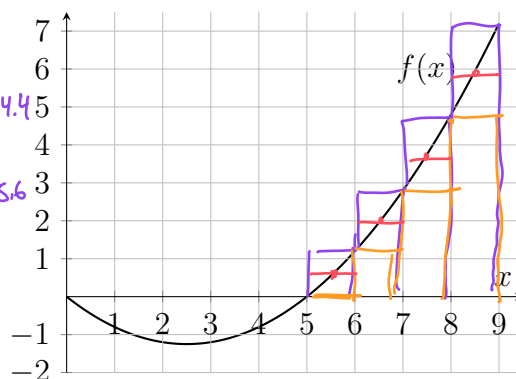
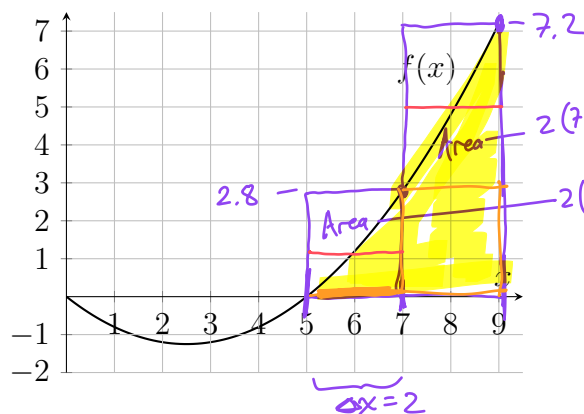
If yes, please figure out the answer! If not, can you give a good estimate?



4. **Group chat:** What information would be nice to have so that you can give a better answer to #3?

Some Left/Right Practice

Here we will consider the graph of $f(x) = \frac{1}{5}x^2 - x$, between $x = 5$ and $x = 9$.



right
midpoints
left

5. Use right endpoints to draw *two* rectangles that approximate the desired area.

(a) What is the width of each rectangle?

☞ This is called Δx .

(b) What is the approximate sum of the areas of these two rectangles? 20

☞ This is called a **right-hand Riemann sum**.

(c) **Group chat:** Is your sum an *under*-estimate or an *over*-estimate of the actual area? Why?

6. On the other graph above, draw another right-hand Riemann sum between $x = 5$ and $x = 9$ with $\Delta x = 1$.

(a) How many rectangles do you end up with? 4

(b) What is the approximate sum of the areas of these rectangles? about 15.9 or 16

(c) Which of your two estimates is closer to the actual area? this estimate

(d) **Group chat:** What could you do to get an even better estimate?

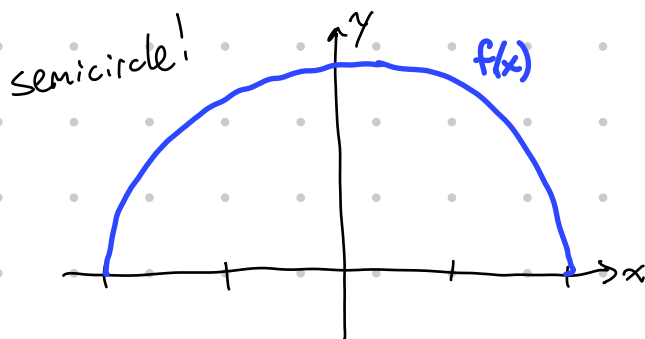
more rectangles!

7. **Group chat:** How would your estimates change if you use *left* endpoints to determine the heights of your rectangles?

☞ This is called a **left-hand Riemann sum**

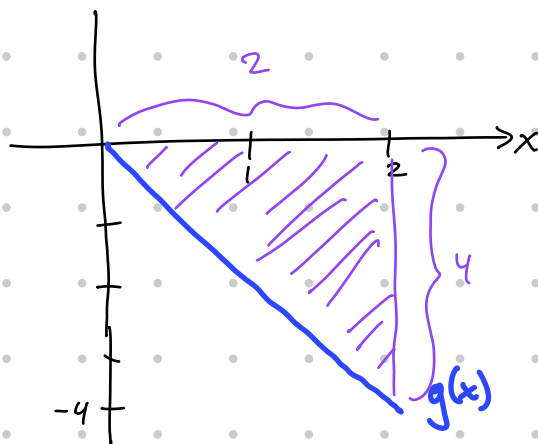
Examples:

$$f(x) = \sqrt{4-x^2}, \quad x \text{ from } -2 \text{ to } 2$$



$$\int_{-2}^2 f(x) dx = \pi(2)^2 \cdot \frac{1}{2} = 2\pi$$

$$g(x) = -2x, \quad x \text{ from } 0 \text{ to } 2$$



$$\int_0^2 -2x dx = -4$$

Area below the
x-axis counts negative!