Calculus II – Day 2

Warm-up

1. Chloe was driving at a *constant* rate of 30 miles per hour for 2 hours.

Group chat: How far did she go in the 2 hours? Make sure you can all explain exactly why the answer is what it is.

What is 'group chat'? Discuss the question and what you think the answer is. If anyone has a different idea or answer (whether right or wrong), discuss that respectfully! Everyone will learn something and come out of it more knowledgeable.

2. Suppose you know that Chloe started driving at 30 miles per hour, but at the end of the first hour, she was traveling 60 miles per hour.

Group chat: Can you figure out *exactly* the distance that Chloe traveled in the 2 hours?

If yes, please figure out the answer! If not, can you give a good estimate?

3. Now Chloe decided to drive for 8 hours!
At the beginning, she was traveling 30 miles per hour.
At the end of 2 hours, she was traveling 60 miles per hour.
At the end of 4 hours, she was traveling 10 miles per hour.
At the end of 6 hours, she was traveling 70 miles per hour.
At the end of 8 hours, she was traveling 20 miles per hour.

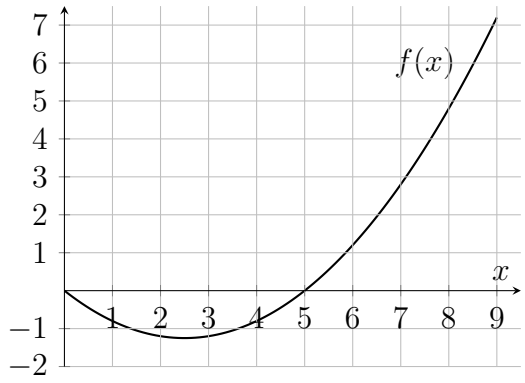
Group chat: Can you figure out *exactly* the distance she traveled in the 8 hours?

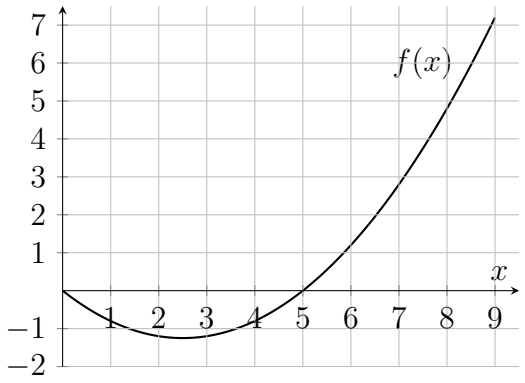
If yes, please figure out the answer! If not, can you give a good estimate?

4. Group chat: What information would be nice to have so that you can give a better answer to #3?

Some Left/Right Practice

Here we will consider the graph of $f(x) = \frac{1}{5}x^2 - x$, between $x = 5$ and $x = 9$.





5. Use right endpoints to draw *two* rectangles that approximate the desired area.

(a) What is the width of each rectangle?

(b) What is the approximate sum of the areas of these two rectangles?

(c) **Group chat:** Is your sum an *under*-estimate or an *over*-estimate of the actual area? Why?

This is called Δx.

This is called a **right-hand Riemann sum**.

6. On the other graph above, draw another right-hand Riemann sum between $x = 5$ and $x = 9$ with $\Delta x = 1$.

(a) How many rectangles do you end up with?

(b) What is the approximate sum of the areas of these rectangles?

(c) Which of your two estimates is closer to the actual area?

(d) **Group chat:** What could you do to get an even better estimate?

7. **Group chat:** How would your estimates change if you use *left* endpoints to determine the heights of your rectangles?

This is called a **left-hand Riemann sum**

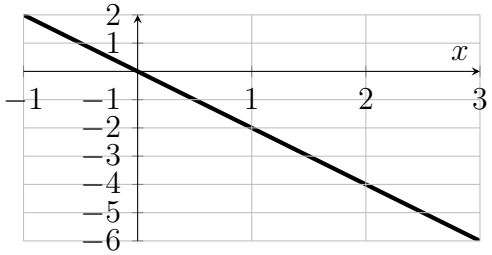
Exact Areas

8. (a) Draw a graph of $f(x) = 1 + x$ and shade the region whose area is $\int_0^3 1 + x\,dx$.

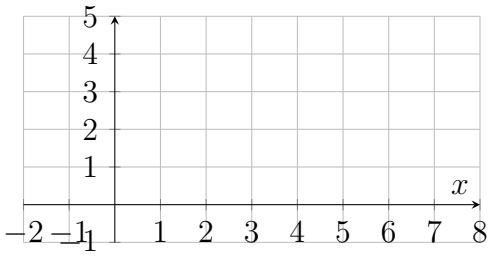
(b) Use ONLY geometry to find the exact answer for $\int_0^3 1 + x\,dx$.

Is your geometry knowledge a little bit rusty?

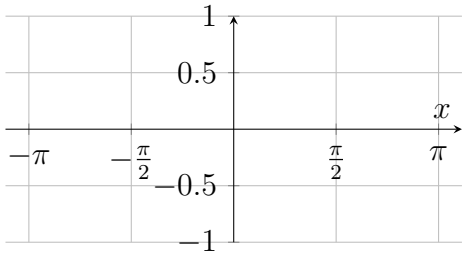
9. Use geometry to find the exact answer to $\int_0^2 -2x\,dx$.



10. Use geometry to find the exact answer to $\int_{-2}^8 4\,dx$.



11. Use geometry to find the exact answer to $\int_{-\pi}^{\pi} \sin(x)\,dx$.



12. (Spicy) Find the exact answer to $\int_{-2}^{2} \sqrt{4-x^2}\,dx$.

Try graphing. Look familiar?

13. Suppose $v(t)$ is the velocity of an object and $s(t)$ is the position of that object at time t.

(a) What is the relationship between the functions $v(t)$ and $s(t)$?

The calculus relationship...

(b) In terms of the function $s(t)$, what is $\int_a^b v(t)\,dt$ calculating?

If you're having trouble using math notation, try using your words first.

(c) Let's try to combine the previous two parts! Fill in the missing pieces of the equation without using the letter v anywhere:

$$\int_a^b v(t)\,dt =$$