Calculus II – Day 3

1. Pollution is removed from a lake on day t at a *rate* of $f(t)$ kilograms per day.

☞ The most important word here is RATE.

(a) **Group chat:** Given the context stated above, what is the meaning of the following statement?

☞ Include units in your answer.

$$f(12) = 500$$

At $t = 12$ (the beginning of day 12), pollution is being removed at a rate of 500 kilograms per day.

(b) **Group chat:** Given the context stated above, what is the meaning of the following statement?

☞ What happens when we find the area under the graph when the graph is a rate of change?

☞ Include units in your answer.

$$\int_5^{15} f(t)\,dt = 4000$$

From $t=5$ to $t=15$, the amount of pollution removed is 4000 kg.

2. A car is driving east with velocity (in miles per hour) at time t given by $v(t)$.

(a) **Group chat:** What does $v(2) = -20$ mean? Use words to explain!

At time $t=2$ (hours), the car is moving towards the west at 20 miles per hour.

(b) Suppose the *position* of the car (east of the start) at time t is given by $s(t)$. Use $s(t)$ to describe what is calculated by

$$\int_{t_1}^{t_2} v(t)\,dt. = s(t_2) - s(t_1)$$

This is the net change in position from time t_1 to time t_2

(c) **Calculus I recall time:** How are $s(t)$ and $v(t)$ related?

☞ Ahhhhhh!

$v(t)$ is the derivative of $s(t)$

$s'(t) = v(t)$

3. Group chat: Why do parts (b) and (c) of #2 indicate that the following is true:

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

The accumulation (integral) of a rate of change of some quantity is the net change in that quantity.

4. Circle the antiderivative(s) of $f(x) = 3x^2$.

How many are there?

$\frac{1}{3}x^3$ $\quad x^3$ $\quad x^3+17$ $\quad x^3+x-17$ $\quad \frac{3}{2}x$ $\quad x^3-2$ $\quad 6x$ $\quad -3x^{-3}$ $\quad x^3+e$

5. Complete the following table.

$f(x)$	What power of x is this?	$\int f(x)\,dx$	$f(x)$	What power of x is this?	$\int f(x)\,dx$
$f(x) = x$	1	$\frac{1}{2}x^2 + C$	$f(x) = \frac{1}{x^5}$	-5	$\frac{-1}{4}x^{-4} + C$
$f(x) = x^2$	2	$\frac{1}{3}x^3 + C$	$f(x) = x^{2/3}$	$\frac{2}{3}$	$\frac{3}{5}x^{5/3} + C$
$f(x) = x^3$	3	$\frac{1}{4}x^4 + C$	$f(x) = x^{-1/2}$	$\frac{-1}{2}$	$2x^{1/2} + C$
$f(x) = x^{10}$	10	$\frac{1}{11}x^{11} + C$	$f(x) = \sqrt{x}$	$\frac{1}{2}$	$\frac{2}{3}x^{3/2} + C$
$f(x) = x^{-3}$	-3	$\frac{-1}{2}x^{-2}$	$f(x) = \frac{1}{x}$	-1	$\ln(x) + C$

6. Find the following antiderivatives:

Remember, you're undoing the derivative.

$$\int \cos(x)\,dx \qquad \int \sin(x)\,dx \qquad \int e^x\,dx \qquad \int 2^x\,dx \qquad \int e^x + x^2\,dx$$

$\sin(x) + C$ $\qquad -\cos(x) + C$ $\qquad e^x + C$ $\qquad \frac{1}{\ln(2)}2^x + C$ $\qquad e^x + \frac{1}{3}x^3 + C$

7. Use the Fundamental Theorem of Calculus to find the *exact* answers:

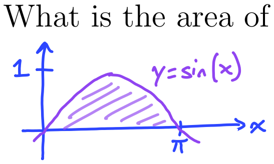
We don't need rectangles anymore!

$$\int_1^2 e^x\,dx \qquad\qquad \int_a^b e^x + x^2\,dx$$

$$\left[e^x\right]_{x=1}^{x=2} = e^2 - e^1$$

$$\left[e^x + \frac{1}{3}x^3\right]_{x=a}^{x=b} = e^b + \frac{1}{3}b^3 - \left(e^a + \frac{1}{3}a^3\right)$$

8. What is the area of exactly one "bump" on the graph of $y = \sin x$?



$$\text{Area} = \int_0^\pi \sin(x)dx = \left[-\cos(x)\right]_{x=0}^{x=\pi} = -\cos(\pi) - (-\cos(0)) = -(-1) - (-1) = 2$$

9. Find the following antiderivatives:

$$\int e^{4x} \cdot 4\,dx \qquad \int e^{4x}\,dx \qquad \int (\sin x)^7 \cos x\,dx$$

$e^{4x} + C$ $\qquad \frac{1}{4}e^{4x} + C$ $\qquad \frac{1}{7}(\sin(x))^7 + C$