The Substitution Rule

1. Review: Find the derivative of $f(x) = e^{4x}$.

☞ Chain rule!

Then evaluate: $\int 4e^{4x}\, dx =$ $\int e^{4x}\, dx =$

☞ Hint: exactly what is different here?

2. Review: Find the derivative of $f(x) = \sin(x^2)$. Now evaluate the following:

$\int 2x\cos(x^2)\, dx$ $\int 5x\cos(x^2)\, dx$ $\int 3x^2\cos(x^3)\, dx$

3. With your group, act out the following dialogue:

Lana: I really need to evaluate $\int \sin(x)e^{\cos(x)}\, dx$
Erez: OK. Let's try substitution for this one!
Lana: Why do you think substitution will work? We are supposed to find something to make u and then search for the derivative of u, also. Oh, I guess we can do that!
Erez: Yeeeeeeah! We can let u be $\sin(x)$, and I see its derivative $\frac{du}{dx} = \cos(x)$ is also in there. Easy!
Lana (shaking her head): Erez, Erez, Erez...

(a) Why is Lana shaking her head at Erez?

(b) What would be a more useful choice for u? Why?

After explaining her idea to Erez, Lana and Erez are left with the problem $\int -e^u\, du$.

Lana: I can do that integral!! The final answer is $-e^u + C$. Yes!!
Erez (shaking his head): My turn! Lana, Lana, Lana...

(c) Why is Erez shaking his head at Lana?

4. For each part, everyone should think alone for 20 seconds and not write anything. Then, as a group, have a quick chat about what you might choose for u and **why**. Finally, check to see if you can finish each integral.

Go through all for problems

(a) $\displaystyle\int e^x \cos\left(e^x\right)\, dx$

(b) $\displaystyle\int \frac{e^{\sqrt{x}}+4}{\sqrt{x}}\, dx$

(c) $\displaystyle\int \sqrt{x-4}\, dx$

Hint: $\sqrt{x-4} = \sqrt{x-4}\cdot 1$

(d) $\displaystyle\int xe^{x^2} \cos\left(e^{x^2}\right)\, dx$

5. The beauty of the substitution method for definite integrals is that you can also substitute for the bounds, meaning you *never* have to change back to the original variable. Find each of the following using u-substitution where you substitute u-values for each of the bounds.

(a) $\displaystyle\int_0^1 e^x \cos\left(e^x\right)\, dx$

(b) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}+4}{\sqrt{x}}\, dx$

(c) $\displaystyle\int_5^8 \sqrt{x-4}\, dx$

(d) $\displaystyle\int_0^1 xe^{x^2} \cos\left(e^{x^2}\right)\, dx$

6. If $\displaystyle\int_1^4 g(x)\, dx = 12$, evaluate $\displaystyle\int_0^1 g(3x+1)\, dx$.

7. More practice! Evaluate the following:

(a) $\displaystyle\int_0^3 2xe^{x^2-5}\,dx$

(b) $\displaystyle\int_0^1 \frac{x^2}{2+3x^3}\,dx$

(c) $\displaystyle\int \frac{e^{\ln x}}{x}\,dx$

Is this simpler than it seems?

(d) $\displaystyle\int \sin(2x)\cos(2x)\,dx$

(e) $\displaystyle\int_0^3 x\sqrt{9-x^2}\,dx$

(f) $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

Hint: $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$