Integration by Parts

1. Review:

$$\frac{d}{dx}\left[u(x)v(x)\right] = v(x)u'(x) + u(x)v'(x)$$

What is this formula?

You might have seen this using different letters!

2. (a) **Use the review:** What is the derivative of $f(x) = xe^x$?

$$f'(x) =$$

(b) $\int (e^x + xe^x)\,dx =$ _____

Hint: look very carefully at part (a)

(c) **Ava:** Milo, this is crazy! Look what just happened!
Milo: Uhhh...hi Ava. Too much coffee early in the morning?
Ava: Yes, but look! We can now figure out the antiderivative $\int xe^x\,dx$.
Milo: What?! That is not the problem we just did in part (b).
Group chat: How can you use the information above to solve for $\int xe^x\,dx$?

3. (a) $\int \left[u'(x)v(x) + u(x)v'(x)\right]\,dx =$ _____

(b) $\int u'(x)v(x)\,dx + \int u(x)v'(x)\,dx =$ _____

(c) **Ava:** What we did in #2 was just a specific example.
Milo: Oh yeah, now we can try to solve for $\int u(x)v'(x)\,dx$.
Group chat: What are they talking about? Try to rewrite $\int u(x)v'(x)\,dx$, but using a different integral.

INTEGRATION BY PARTS

$$\int u \cdot dv \quad = \quad u \cdot v \quad - \quad \int v \cdot du$$

Keep in mind the following important pieces of strategy:

- You have to "break up" the integral into two multiplied pieces, one called u and the other called dv.
- Whatever you choose as u will have to have its derivative taken.
- Whatever you choose as v will have to have its antiderivative taken.
- You will have to find the antiderivative of $v \cdot du$ in order to finish.

4. Use integration by parts to find $\int x \sin x \; dx$.

Guess for u: $\Rightarrow$ **Calculate du:**

Guess for dv: $\Rightarrow$ **Calculate v:**

Your u and dv MUST multiply to equal $x \sin x$.

Now calculate: $uv - \int v \, du =$

If you did this right, finding the antiderivative of $v \; du$ should not be harder than the original problem.

5. Marissa: Javier, I really need to know $\int x \, \ln(x) \, dx$.

Javier: OK. I think we should do what we did before, so let's try

$$u = x \qquad \text{and} \qquad dv = \ln(x) \, dx.$$

Marissa: Thanks for trying to help, Javier, but that made it worse.
Group chat: What went wrong with Javier's idea?

Group task: Don't give up! Is there a different way we could choose u and dv? Can you figure out $\int x \, \ln x \, dx$?

6. Find $\int x^3 \ln x \, dx$.

7. Find $\int \ln x \, dx$.

☞ Hint: remember that $\ln x = 1 \cdot \ln x$.

8. Find $\int x^2 \cos x \, dx$.

☞ The integral you get after doing integration by parts won't be simple, but it will be simpler than what you started with. Keep going!

9. Find $\int_0^{\pi} x \sin x \, dx$.

10. Find $\int x^2 e^{x^3}\,dx$.

☜ SLOW DOWN! Is there something easier than integration by parts for this?

11. Spicy! Find $\int e^x \cos x\,dx$.

☜ You will need to do integration by parts twice. After that, you should end up with the same integral you started with, so it might seem hopeless. But is it?

12. Find $\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$

13. Find $\int x^n \ln x\,dx$. Here, assume n is a constant and $n \neq -1$.

☜ Why doesn't $n = -1$ work?