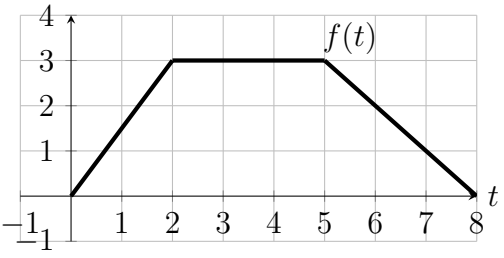
More Integration

1. The graph of a function $f(t)$ is shown below. Let $g(x) = \int_0^x f(t)\,dt$



(a) What are the following values of g?

$g(0) =$ $g(1) =$ $g(2) =$ $g(3) =$

$g(4) =$ $g(5) =$ $g(6) =$ $g(7) =$

(b) **Felix:** Look! $g'(4) = 3$.
Group chat: Is Simon correct? Why or why not?

(c) **Maura:** I need a value of x such that $g'(x) = 2$.
Group chat: Help Maura out. Where is $g'(x) = 2$?

(d) **Simon:** Ah-ha! Now I see that f is the _____ of g!

2. Quick! Find the following derivatives!

$$\frac{d}{dx}\int_0^x t\sin(t)\,dt \qquad \frac{d}{dx}\int_1^x e^{t^2}\,dt \qquad \frac{d}{dx}\int_x^0 |\cos(t)|\,dt$$

3. Now suppose we want to find $\frac{d}{dx}\int_1^{x^2} e^{t^2}\,dt$.

(a) Let $F(t)$ be an antiderivative of e^{t^2}. Express $\int_1^x e^{t^2}\,dt$ in terms of F.

(b) Differentiate what you wrote in part (a) and simplify. What have you found?

4. If $F(x) = \displaystyle\int_0^{x^2} \left(t^2 - 10\right) dt$, what is $F'(x)$?

5. If $F(x) = \displaystyle\int_1^{\ln(x)} (s^6 + e^{4s})\, ds$, what is $F'(x)$?

6. If $h(x) = \displaystyle\int_{3x}^{0} \cos(y)\, dy$, what is $h'(x)$?

7. If $q(x) = \displaystyle\int_{\ln(x)}^{2x} \left(1 + t^2\right) dt$, what is $q'(x)$?

8. The *sine integral function*

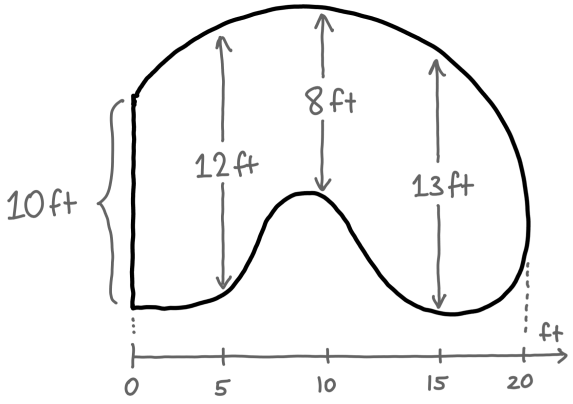
$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

is important in electrical engineering.

(a) Use technology to sketch a graph of $\mathrm{Si}(x)$.

(b) What is the smallest positive value of x at which $\mathrm{Si}(x)$ has a local maximum?

9. Chloe: I need to find the area of my backyard pool. I made a sketch of it and took some measurements:



Erez: I can't find the exact area of your pool, but I can find values M and N such that the area is *between* M and N.

Group chat: What is Erez talking about? Can you find such values M and N?

What is your *best estimate* of the area of the pool? How far off from the actual area could you be?

10. The speed of a runner was measured each second for the first five seconds of a race:

time t (seconds)	velocity v (meters/second)
0	0
1	6.8
2	8.2
3	9.8
4	10.2
5	10.4

(a) Approximately how far did the runner travel in these five seconds?

(b) How far off could your answer in part (a) be from the actual distance that the runner traveled? Can you tell if your estimate is an overestimate or an underestimate?

(c) What additional assumptions could you make about the runner's velocity that would improve your answers in part (b)?