

Taylor and Maclaurin Series

1. Find the sum of each series. (All the series converge and NONE of them are geometric.)

(a) $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

🔖 Hint: Look back and forth between this page and the board.

(b) $1 + (\ln 3) + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \dots$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n 9^n \pi^{2n}}{4^n (2n)!}$

🔖 Note that $9 = 3^2$ and $4 = 2^2$.

2. **Renita:** I really want to find the Maclaurin series for $f(x) = x^2 \cos(x^2)$.

Logan: OK! All we have to do is take the derivative of $f(x)$ over and over and over repeatedly forever, and then plug in $x = 0$ each step along the way.

Group chat: Why Logan's strategy a *very bad* idea?

Renita: I have a faster way, Logan! Instead, let's use the Maclaurin series *we already know*:

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

Logan: Oh...we can do this in two steps! First, we will do the $\cos(x^2)$ piece:

$$\cos(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$$

Group chat: What did Logan just do? What's the next step towards finding the series for $f(x) = x^2 \cos(x^2)$?

3. **Macie:** Hey Logan, I am so happy we just found the Maclaurin series for $f(x) = x^2 \cos(x^2)$.

$$x^2 \cos x^2 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n)!}$$

Logan: Cool! I have another problem now. I want to find both $f^{(17)}(0)$ and $f^{(18)}(0)$.

Renita: Well, you definitely do NOT want to take the derivative of $f(x)$ 17 or 18 times.

Group chat: Why did Renita say this?

Macie: I agree. The information I want is hidden in the Maclaurin Series!

Group chat: How can you use the Maclaurin Series for $f(x) = x^2 \cos(x^2)$ to find $f^{(17)}(0)$ and $f^{(18)}(0)$?

4. In statistics, the function e^{-x^2} comes up as part of the “normal distribution.”

(a) **Group Discussion:** Why is finding $\int_0^1 e^{-x^2} dx$ difficult?

(b) Find the Maclaurin Series for e^{-x^2} .

 Hint: Substitution

(c) Find an *approximation* for $\int_0^1 e^{-x^2} dx$.