Exam 2 Practice Problems

1. Determine whether each of the following statements is *always true*, *sometimes true*, or *never true*. Explain your reasoning.

(a) If $f(x)$ is continuous on $[1, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f(x)\,dx$ converges.

(b) If f' is continuous on $[0, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_0^\infty f'(x)\,dx = -f(0)$.

(c) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

(d) If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

2. Evaluate each integral or show that it is divergent:

(a) $\displaystyle\int_0^\infty \frac{x}{e^x}\,dx$

(b) $\displaystyle\int_1^2 \frac{dx}{x\ln(x)}$

3. Let $a_n = \dfrac{\ln(n)}{\sqrt{n}}$ for each positive integer n.

(a) Does the sequence $\{a_n\}$ converge or diverge? Explain.

(b) Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Explain.

4. Determine if the following series converges or not. If it does, then determine the sum.

(a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{\pi}{3}\right)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+2}}{3^n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n\, n!}$

5. (a) Differentiate the Maclaurin series for $\sin(x)$. Explain how this shows you that $\frac{d}{dx}\sin(x) = \cos(x)$.

(b) Differentiate the Maclaurin series for e^x. Explain how this shows you that e^x is its own derivative.

6. For the following, find the Taylor polynomial of degree n centered at a.

(a) $\sin(x)$ for $a = \frac{\pi}{2}$ and $n = 4$

(b) $\sqrt{1+x}$ for $a = 3$ and $n = 2$

7. Find the interval of convergence of the following series:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{b^n}(x-a)^n$ where $b > 0$

(b) $\displaystyle\sum_{n=0}^{\infty} n!(x-a)^n$

8. The limit $\displaystyle\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$ is tricky to evaluate, but not if you know about Maclaurin series! Use the Maclaurin series for $\cos(x)$ to evaluate the limit.

9. Recall that $\arctan(1) = \frac{\pi}{4}$. Use the Maclaurin series for $\arctan(x)$ to produce a series that converges to π.