

Multivariable Functions

1. Sketch the 3D coordinate axes, and label each axis. Then draw the graphs of the following equations.

$$y = 2$$

$$x^2 + y^2 = 1$$

2. Let $f(x, y) = \sqrt{x^2 - y^2}$. Note that this is a function of *two* variables, x and y !

- (a) What are all x and y values such that $f(x, y) = 0$? Draw the set of these values in the (x, y) -plane.

- (b) What are all x and y values such that $f(x, y)$ is a real number? Draw the set of these values in the (x, y) -plane.

- (c) Choose some points (x, y) and compute $f(x, y)$. Where is $f(x, y)$ big? Where is $f(x, y)$ small?

👉 Everyone at your table can choose different points! Then compare answers.

- (d) Sketch the graph of $f(x, y)$ on the 3D coordinate axes.

3. (a) What is the distance between $(0, 0)$ and $(1, 1)$ in 2D?

🔍 Is there a right triangle hiding here?

(b) What is the distance between $(0, 0, 0)$ and $(1, 1, 1)$ in 3D?

🔍 Where is the right triangle?

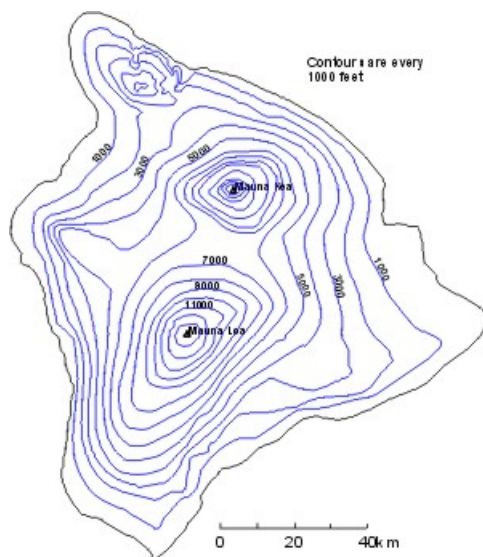
4. Consider the points $P = (1, -2, 1)$, $Q = (5, 1, 1)$, and $R = (1, 1, 1)$.

(a) Find the distance between the points P and Q .

(b) Find an equation of the sphere with radius 3 centered at P .

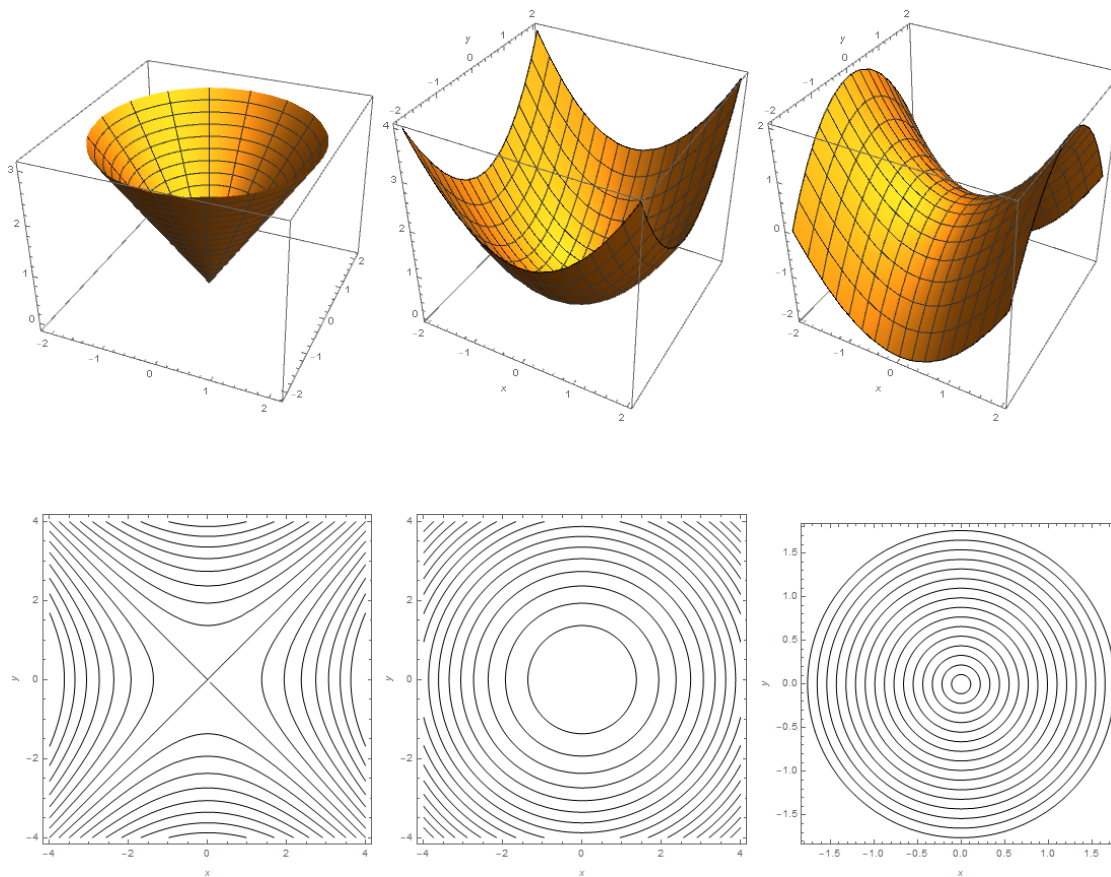
(c) Does R lie within the sphere of radius 2 centered at P ?

5. The image at below is a contour map of the big island of Hawaii. It shows level curves on the surface of the island.

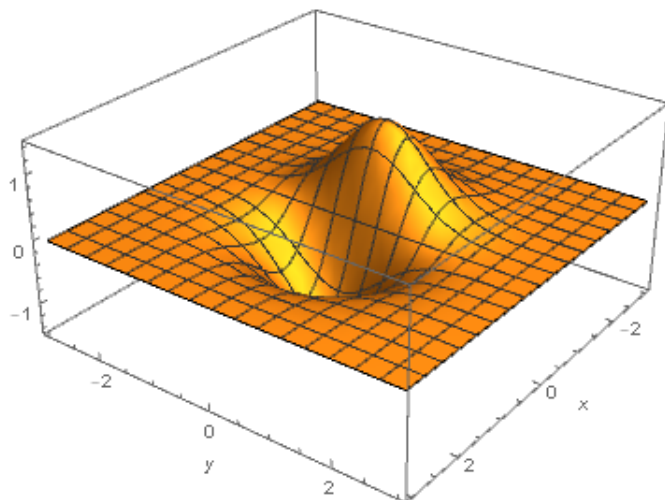


- (a) Where is a (mostly) flat spot on the island?
- (b) Where are some steep places on the island?

6. Match the following 3D graphs with their level curves.



7. The graph of $f(x, y) = -3xe^{-x^2-y^2}$ is pictured below.



(a) Sketch the level curves of f .

(b) Suppose you fix $x = 0$. What does the cross-section of the graph look like for $x = 0$?
This is called the $x = 0$ *trace* of the graph.

(c) Now fix $y = 0$. What does the $y = 0$ trace look like?