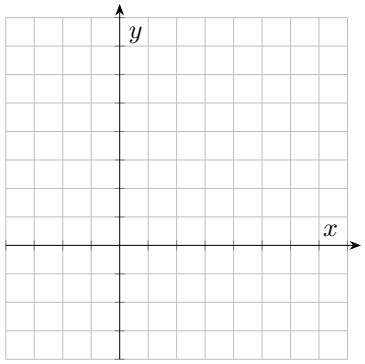
Lines and Planes

1. **Group investigation**: Pick several different values of t and plot the points whose x and y coordinates are given by:

$$\begin{cases} x = 3 - 2t \\ y = 1 + 3t \end{cases}$$

Everyone at your table can pick different values of t. Then share your answers.



What do you notice?

2. **True or False:** In two dimensions, a line is determined by:

   (a) Two different points on the line.

   (b) One point on the line and the slope of the line.

   (c) One point on the line and a vector parallel to the line.

3. **True or False:** In *three* dimensions, a line is determined by:

   (a) Two different points on the line.

   (b) One point on the line and the slope of the line.

   (c) One point on the line and a vector parallel to the line.

4. Write equations for the line passing through the point $(4, 7, -2)$ and parallel to the vector $\langle 3, -1, 2 \rangle$...

   (a) In vector form:

   (b) In parametric form:

5. True or False: In three dimensions, a *plane* is determined by:

Make drawings, use your hands, use props, etc.!

(a) Three points (not all in the same line)

(b) One point and two different (non-parallel) direction vectors

(c) One point and a perpendicular vector

6. Let's think about the plane P that goes through the points $A = (1, 0, 0)$, $B = (0, 2, 0)$, and $C = (0, 0, 3)$.

(a) Make a sketch of the plane P.

(b) One vector parallel to the plane is $\overrightarrow{AB} = \langle -1, 2, 0 \rangle$. A different vector parallel to P is $\overrightarrow{AC} = \langle -1, 0, 3 \rangle$. How can you find a vector $\mathbf{n}$ that is *perpendicular* to the plane P?

Hint: You have TWO vectors that are parallel to the plane.

(c) Suppose that $R = (x, y, z)$ is a random point located in plane P. Explain why the vector $\overrightarrow{RA} = \langle x - 1, y - 0, z - 0 \rangle$ is parallel to the plane P.

(d) We are going to find $\mathbf{n} \cdot \overrightarrow{RA}$ in two ways:

- Compute $\mathbf{n} \cdot \overrightarrow{RA}$ using the definition of the dot product.
- Since $\overrightarrow{RA}$ is parallel to P and $\mathbf{n}$ is perpendicular to P, what is $\mathbf{n} \cdot \overrightarrow{RA}$?

(e) **Milo:** "WOW! That means an *equation* for the plane P is $6(x - 1) + 3(y - 0) + 2(z - 0) = 0$."
Group chat: How did Milo arrive at this equation?

(f) Explain why $6(x - 0) + 3(y - 2) + 2(z - 0) = 0$ is another way to write an equation for plane P.

(g) What is the *general strategy* for finding the equation of a plane?

7. Suppose that P is the plane that passes through the points $A = (-1, 2, 3)$, $B = (0, 4, 4)$, and $C = (-1, 2, 4)$. Find an equation for P.

8. *Spicy*: Find the angle between the planes $x + y + z = 1$ and $x - y + 3z = 3$.