Second Partial Derivatives

1. Sketch the graph of a function $z = f(x, y)$ whose derivatives f_x and f_y are always positive. Can you give an example of formula for such a function?

2. Sketch the graph of a function $z = f(x, y)$ whose derivative f_x is always negative and whose derivative f_y is always positive. Can you give an example of formula for such a function?

3. Let $f(x, y) = x^2 + 4xy + y^2 - 4x + 16y + 3$. Find all values of x and y such that $f_x(x, y) = 0$ and $f_y(x, y) = 0$ simultaneously. How would you describe the graph of $f(x, y)$ at these points?

4. **Chloe:** In Calculus I, we learned about more than just the first derivative $f'(x)$. We also learned about the second derivative $f''(x)$.

Bastian: I bet there are second derivatives in Calculus II, too! If you start with a function $f(x, y)$, you can take a partial derivative and then take another partial derivative!

Group chat: If you start with a function $f(x, y)$, how many *different ways* can you take a partial derivative and then take another partial derivative?

☞ There is more than one way!

Chloe: OK, so if you take the partial derivative of $f(x, y)$ with respect to x, and then take the partial derivative again with respect to x, what does that tell you about f?

Group chat: How would you answer Chloe's question?

Bastian: That makes sense. But what do the *other* second partial derivatives tell you about f?

Group chat: How would you answer Bastian's question?

5. Find *all* second partial derivatives of $f(x, y) = 3xy^2 - 2y + 5x^2y^2$. Does anything surprise you about these partial derivatives?

6. Find *all* second partial derivatives of $g(x, y) = \sqrt{x^2 + y^2}$. Does anything surprise you about these partial derivatives?

7. Find *all* second partial derivatives of $h(x, y) = e^x \tan(y)$. Does anything surprise you about these partial derivatives?

8. How many second partial derivatives are there for the function $f(x, y, z) = ye^x + x\ln(z)$? Which ones do you think will be equal? Then compute these partial derivatives to check your hypothesis!