Second Partial Derivatives

1. Sketch the graph of a function z = f(x, y) whose derivatives f_x and f_y are always positive. Can you give an example of formula for such a function?

2. Sketch the graph of a function z = f(x, y) whose derivative f_x is always negative and whose derivative f_y is always positive. Can you give an example of formula for such a function?

3. Let $f(x,y) = x^2 + 4xy + y^2 - 4x + 16y + 3$. Find all values of x and y such that $f_x(x,y) = 0$ and $f_y(x,y) = 0$ simultaneously. How would you describe the graph of f(x,y) at these points?

4. Chloe: In Calculus I, we learned about more than just the first derivative f'(x). We also learned about the second derivative f''(x).

Bastian: I bet there are second derivatives in Calculus II, too! If you start with a function f(x, y), you can take a partial derivative and then take another partial derivative!

Group chat: If you start with a function f(x,y), how many different ways can you take a partial derivative and then take another partial derivative?

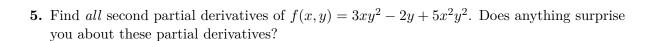
There is more than one way!

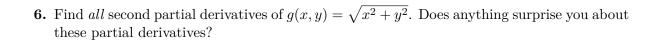
Chloe: OK, so if you take the partial derivative of f(x, y) with respect to x, and then take the partial derivative again with respect to x, what does that tell you about f?

Group chat: How would you answer Chloe's question?

Bastian: That makes sense. But what do the *other* second partial derivatives tell you about f?

Group chat: How would you answer Bastian's question?





7. Find all second partial derivatives of $h(x, y) = e^x \tan(y)$. Does anything surprise you about these partial derivatives?

8. How many second partial derivatives are there for the function $f(x, y, z) = ye^x + x \ln(z)$? Which ones do you think will be equal? Then compute these partial derivatives to check your hypothesis!