Tangent Planes

1. Group Conjecture: What is the equation for the plane tangent to $z = f(x, y)$ when $(x, y) = (x_0, y_0)$?

Look at the equation for a tangent line on the wall. Then fill in the blanks here.

$$z = \_\_\_\_\_ + \_\_\_\_\_\left(x - \_\_\_\_\_\right) + \_\_\_\_\_\left(y - \_\_\_\_\_\right)$$

2. Find the equation of the plane that is tangent to $f(x, y) = 2x^2 + y^2 - 3y$ at $(x, y) = (1, 1)$.

3. Erez: Hey! The tangent plane is really close to the original function for pairs of (x, y) that are "close" to $(1, 1)$.

Jade: Oh, so we can approximate $f(1.1, 0.9)$ without actually using $f(x)$.

Group task: Help Jade approximate $f(1.1, 0.9)$ without actually using $f(x)$.

4. Use a tangent plane to estimate the value of $g(x, y) = x \sin(x + y)$ at $(x, y) = (0.5, 3)$.

5. Find the equation of the plane tangent to $f(x, y) = 3 - 2x + 5y$ at $(x, y) = (2, 3)$. Simplify as much as possible.

Does the result surprise you?

6. Find the linearization of $f(x, y) = \sqrt{10 - x^2 - 2y^2}$ at $(2, 1)$ and use it to approximate $f(1.9, 1.1)$.

7. Suppose that your friend claims that the equation of the tangent plane to the graph of $f(x, y) = x^3 - y^2$ at the point $(4, 5)$ is

$$z = 39 + 3x^2(x - 4) - 2y(y - 5).$$

(a) Why is this not possibly the equation of a tangent plane?

(b) What mistake did your friend make?

(c) What is the correct equation of the tangent plane?

8. Find the linearization of $z = y \ln(x)$ at $(1, 4, 0)$.