Double Integrals

1. *Warm-up*: Standing outside of Buntrock Commons on a snowy day, suppose you measure the **rate** (in cm per hour) at which snow is falling every 30 minutes. Here is the data:

Time of Day	12:00	12:30	1:00	1:30	2:00	2:30	3:00
Rate of snowfall (cm per hour)	1	2	3	1	0	2	3

(a) **Group chat:** How can you estimate the actual number of centimeters of snow that fell between 12:00 and 12:30?

(b) **Group chat:** What is a good estimate for the actual number of centimeters of snow that fell between 12:00 and 3:00?

☝ There is more than one way to estimate this!

2. Now suppose the snow fell at the *same* rate all throughout Northfield (that is, the rate does not change based on location). For simplicity, assume that Northfield is a one square kilometer region (that is, 100,000 × 100,000 cm).

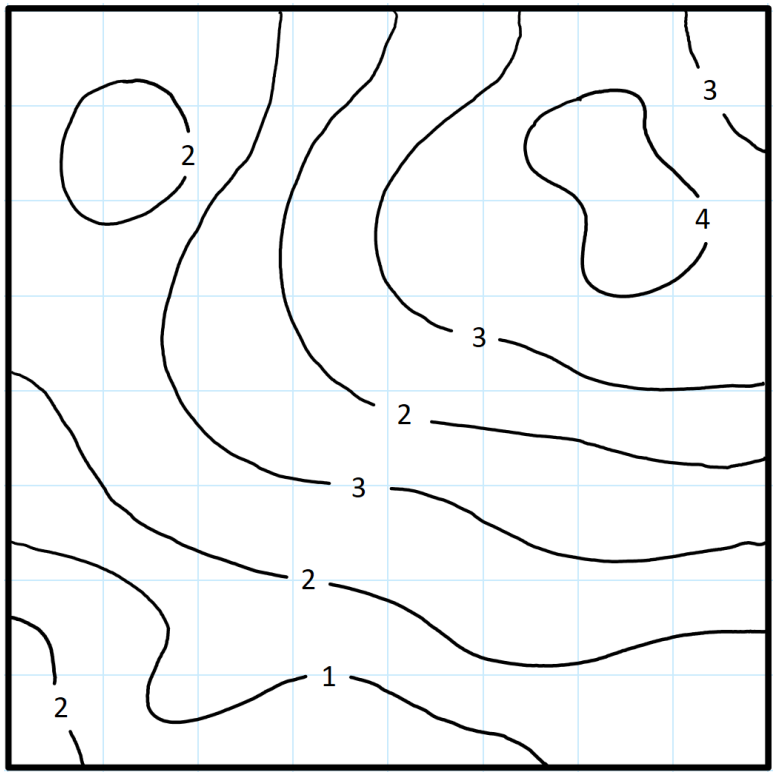
Group chat: What is a good estimate for the actual volume of snow (in cm^3) collected in Northfield?

☝ You can measure volume in cm^3.

3. Now suppose that snow is falling at a rate of 3 cm per hour on the St. Olaf half of Northfield and at a rate of 2 cm per hour on the Carleton half of Northfield.

Group chat: What is a good estimate for the volume of snowfall in Northfield in 1 hour?

4. Now suppose the rate at which snow falls is *different* throughout Northfield. Here is a contour map of the different rates in a 1 km square region at 12:00:



(a) **Group chat:** How can you estimate the volume of snow that fell within this region between 12:00 and 12:30?

(b) **Group chat:** Whatever you did in part (a), what could you do to improve your estimate?

5. Sketch the solid under the graph of $f(x, y) = 5$ and above the region $R = [0, 3] \times [0, 2]$. Then evaluate the following integral.

R is the set of points (x, y) such that $0 \leq x \leq 3$ and $0 \leq y \leq 2$.

$$\iint_R 5 \, dA$$

6. Sketch the solid under the graph of $f(x, y) = 3 - x$ and above the region $R = [0, 3] \times [0, 2]$. Then evaluate the following integral.

$$\iint_R (3 - x) \, dA$$

7. Sketch the solid under the graph of $f(x, y) = \sqrt{4 - x^2}$ and above the region $R = [-2, 2] \times [0, 3]$. Then evaluate the following integral.

$$\iint_R \sqrt{4 - x^2}\, dA$$

8. Sketch the solid between the graph of $f(x, y) = 1 - y$ and the region $R = [0, 2] \times [-2, 2]$. Then evaluate the following integral.

$$\iint_R (1 - y)\, dA$$

☜ Why does this problem say "between" instead of "under"?