Final Exam Guidance

MATH 126 A/B • Fall 2025

The following is intended to help you focus your studying for the Final Exam. The exam will be cumulative, but will emphasize topics that we studied in the last month or so—especially partial derivatives and double integrals. Of course, these topics require understanding of previous topics, especially derivatives, integrals, and 3D geometry. The exam will also include questions on topics from earlier in the course. The following notes might not encompass everything you could see on the exam, but they are intended to help you think about the course content and study effectively.

Important Notes

- You may use one side of one sheet of notebook paper — letter size (8.5×11 inches) or A4 size (210×297 mm) — containing notes that you prepare in advance.
- Calculators will not be helpful and definitely *not necessary*. This exam will test calculus concepts, not your ability to do arithmetic. You will not be required to simplify arithmetic. You *may* use a calculator, but only simple arithmetic, and you must state where exactly you used your calculator. You may not use a phone, computer, or internet-capable device.
- As you study, focus on fluency and deep understanding of calculus concepts rather than only pure memorization.

What should I know and be able to do?

Derivatives and other prerequisites

- Understand derivatives as instantaneous rate of change.
- Differentiate common functions, such as $f(x) = c$, $f(x) = x^n$, $f(x) = e^x$, $f(x) = \ln(x)$, $f(x) = \sin(x)$, $f(x) = \cos(x)$, $f(x) = \tan(x)$.
- Be able to sketch the graphs of the previous functions.
- Apply the constant multiple, sum/difference, product, quotient, and chain rules for derivatives.
- Understand the basics of taking limits. In particular as n increases, logarithms are less than polynomials, polynomials are less than exponentials, and exponentials are less than factorials. Within the realm of powers, higher powers win. Within the realm of exponentials, higher bases win. If you are taking the limit of a fraction, and the numerator and denominator "balance," then you only use the coefficients of the dominant terms

Integrals

- Understand that a definite integral represents accumulated change if the function being integrated is a rate of change.
- Understand that a definite integral also calculates (signed) area.

- Know the following antiderivatives: $\int c \ dx$, $\int x^n \ dx$, $\int \frac{1}{x} \ dx$, $\int e^x \ dx$, $\int \sin(x) \ dx$, $\int \cos(x) \ dx$
- Be able to apply properties of integrals — for example: sums, constant multiples, flip the bounds of integration.
- Use substitution and integration by parts to evaluate integrals.
- Set up and evaluate integrals representing the area between the graphs of two functions, the length of the graph of a function, and a volume of revolution.
- Be able to apply the fundamental theorem of calculus (both versions).
- Identify improper integrals and determine whether they converge or diverge.

Sequences and Series

- Explain what a sequence is, what it means for a sequence to converge or diverge, and how to determine whether a given sequence converges or diverges.
- Be able to give examples of convergent and divergent sequences.
- Explain what a series is, what it means for a series to converge or diverge (this involves *partial sums* of the series).
- Explain how a series is different from a sequence.
- Identify and give examples of geometric series, and evaluate finite and infinite geometric sums.
- Use the integral test to determine whether a series converges or diverges.
- Use the limit comparison test to determine whether a series converges or diverges.
- Use the ratio test to determine whether a series converges or diverges.
- Explain what a power series is, and identify the center and radius of convergence of a power series.
- Find power series (specifically, a Taylor or Maclaurin series) that converges to a given function. Be able to use such a series to approximate the function near a point.
- Use substitution, derivatives, and antiderivatives to modify a given power series, obtaining a new power series that converges to a function.

Multivariable Functions

- Understand how a function of two variables relates to its graph and contour plot.
- Describe what a vector is, and what it means for two vectors to be equal.

- Perform operations on vectors, including addition and scalar multiplication.
- Compute the dot product of two vectors, and state how the dot product relates to the angle between two vectors.
- Compute the cross product of two vectors, and use the cross product to find a vector orthogonal to two given vectors.
- Determine the equation of a line between two points in space, in both parametric and vector form.
- Determine the equation of a plane in space from a set of criteria (such as three given points, or a point and a normal vector).
- Compute partial derivatives. Interpret a partial derivative as rates of change in a particular direction.
- Find the tangent plane to the graph of a 2-variable function at a given point. Use the tangent plane linearization to approximate values of the function near the given point.
- Compute the gradient and directional derivatives, and interpret them as rates of change in a particular direction.
- Understand double integrals as signed volume under the graph of a surface.
- Convert a double integral over a rectangular or triangular region to an iterated integral (that is, an integral with two sets of bounds and a $dydx$ or $dxdy$).
- Evaluate iterated integrals by evaluating the inner integral, then evaluating the outer integral.

Common mistakes to watch out for

- It may be tempting to try to combine antiderivatives with derivative rules. There are no easy "rules" for antidifferentiation, only techniques! In particular, there is no such thing as a product rule or chain rule for antiderivatives.
- Small algebra mistakes happen. We'll learn from them but move on. Be careful not to make *fundamental* algebra mistakes. Square roots and other powers can't be distributed on addition, and neither can sine, cosine, or logarithms. For example:

$$\sqrt{x^2+9} \quad \text{IS NOT} \quad x+3$$
$$(x+4)^3 \quad \text{IS NOT} \quad x^3+4^3$$
$$\cos(a+b) \quad \text{IS NOT} \quad \cos a+\cos b$$
$$\ln(a+b) \quad \text{IS NOT} \quad \ln a+\ln b$$
$$\frac{a+b}{a+c} \quad \text{IS NOT} \quad \frac{b}{c}$$

- People often *overuse* natural logs for antiderivative. Just because you see a fraction "$\frac{1}{\text{something}}$" does not mean natural log is even part of the answer. For example:

$$\int \frac{1}{x^2+4x+4}\,dx \quad \text{IS NOT} \quad \ln|x^2+4x+4|+C$$
$$\int \frac{1}{\text{something}}\,dx \quad \text{IS NOT} \quad \ln|\text{something}|+C$$

- When performing a u-substitution on a definite integral, make sure you *either* update the bounds to the new variable *or* substitute back to the original variable. In particular:

$$\int_0^2 2x(x^2+1)^3\,dx \quad \text{IS NOT} \quad \int_0^2 u^3\,du.$$

- If a series converges, then its terms must go to zero, but the reverse statement is not true. In other words, the Divergence test never lets you conclude that a series *converges*. Specifically:

$$\text{if } \lim_{n\to\infty} a_n = 0, \quad \text{IT DOES NOT MEAN} \quad \sum_{n=0}^{\infty} a_n \text{ converges.}$$

How should I study?

First, understand that people learn differently and process information in different ways and at different speeds. I suggest:

- Read through each section of *Active Calculus* that we have studied and think about whether or not the main ideas make intuitive sense. Can you explain them out loud?

- The problems in each section of *Active Calculus* are great practice problems. There is also document containing practice problems on the course website and another large set of practice problems on Edfinity.

- Do a few problems each day, ramping up as we get closer to the exam day. Talk with your classmates about the problems and see if you can explain the solutions to each other. Talk with Prof. Wright you want to make sure things are correct or if you are uncertain about anything.

- Work on fluency! The exam is timed and you want to be able to do the problems efficiently. Perhaps give each other a few selected problems from a few different sections and time yourselves. This way, you won't know which section the problem came from.

- COME VISIT PROF. WRIGHT AND ASK QUESTIONS.