

Written Homework 9

MATH 126

Solve each of the following problems. Work out your problems on scratch paper first, then write your solutions neatly on the pages you plan to turn in. Write the problems in assigned order, with each problem clearly labeled. Use words to clearly explain your work and methods. The reader should never have to guess or infer your intentions.

For a brief guide to writing homework solutions, see *Writing Mathematics Well* from Harvey Mudd College.

Scan or photograph your solutions and submit them (as a single file) to the Written Homework 9 assignment on Moodle. This assignment is due at classtime on **Monday, October 27**.

1. Consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 5^n}$. Note that this power series is centered at $x = 0$.
 - (a) Find the radius of convergence r of this series. That is, find the value r such that this series converges for $|x| < r$ and diverges for $|x| > r$. Explain your work.
 - (b) Now test whether the series converges at the *endpoint* of the interval of convergence. That is, determine whether the series converges at $x = r$ and also at $x = -r$.
2. Suppose that $\sqrt{1+x} = \sum_{n=0}^{\infty} c_n x^n$. Find the first four coefficients c_0 , c_1 , c_2 , and c_3 for this power series.
3. Find the Maclaurin series for each of the following functions. Be sure to give a general formula for the n th term of each series.
 - (a) $f(x) = \cos(2x)$
 - (b) $f(x) = x^2 e^x$
4. Let $g(x) = \arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ (note this is only true for $-1 \leq x \leq 1$).
 - (a) Find $g^{(41)}(0)$. You *don't* need to differentiate $\arctan(x)$ 41 times, but you *do* need to explain your reasoning.
 - (b) Find $g^{(42)}(0)$. Again, you *don't* need to differentiate $\arctan(x)$ 42 times, but you *do* need to explain your reasoning.