Linear Algebra – Day 35

MATH 220

Here are sets of "vectors" along with definitions for "addition" and "scalar multiplication." For each, give an example of a vector. Then decide whether the set is closed under addition and scalar multiplication. Is there a "zero vector"? Do the other properties hold? If not, which fail? Finally, decide if each is a vector space.

V, +, and ·	**Vector example**	**Closed under +?**	**Zero vector 0?**	**Closed under ·?**	**Other properties?**	**Vector space?**
$\mathbb{R}^3$ with usual + and ·	$\begin{bmatrix} 4 \\ -13 \\ 2.5 \end{bmatrix}$	yes	$\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	yes	yes	yes
$\mathbb{R}^4$ with usual + and ·						
$\mathbf{P}^2$: polynomials of degree ≤ 2 (variable is x), with polynomial + and scalar ·	$4x^2-13x+2.5$	yes	$\mathbf{0}$ is the number 0	yes	yes	yes
all $\begin{bmatrix} x \\ y \end{bmatrix}$ in $\mathbb{R}^2$ where $x, y \geq 0$, with same + and · as $\mathbb{R}^2$						
$\mathbb{R}^{2\times 2}$: all 2×2 matrices, with usual matrix + and scalar ·						
$\mathbb{R}^{3\times 2}$: all 3×2 matrices, with usual matrix + and scalar ·						
all polynomials of degree *exactly* 2 (variable is x), with polynomial + and scalar ·						

V, +, and $\cdot$	**Vector example**	**Closed under +?**	**Zero vector 0?**	**Closed under $\cdot$?**	**Other properties?**	**Vector space?**
upper-triangular 2×2 matrices with matrix + and scalar $\cdot$						
$\mathbb{R}^+$: all positive real numbers, "$\mathbf{r} + \mathbf{s}$" is done by taking $\mathbf{rs}$, "$c \cdot \mathbf{r}$" is done by taking $\mathbf{r}^c$						
$\mathbf{P}^n$: polynomials of degree $\leq n$ (variable is x), with polynomial + and scalar $\cdot$						
$\mathbb{R}^{m \times n}$: all $m \times n$ matrices, with usual matrix + and scalar $\cdot$						
$\mathbf{P}$: all polynomials (variable is x) with polynomial + and scalar $\cdot$						
$\mathcal{C}$: all continuous functions (real valued functions $f(x)$), with ordinary function + and scalar $\cdot$						