

Math 234

Logical Statements

Day 3

Discuss the following problems with the people at your table.

1. **Warm-Up:** Write the converse, inverse, and contrapositive of the statement:

If it is hot outside, then I will buy ice cream.

(a) Converse:

(b) Inverse:

(c) Contrapositive:

2. Complete the following truth table.

\leftrightarrow denotes
biconditional

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T			
T	F			
F	T			
F	F			

How does $p \leftrightarrow q$ relate to $p \rightarrow q$ and $q \rightarrow p$?

3. What is the negation of the biconditional statement $p \leftrightarrow q$? (Consider extending the truth table above.)

4. Express the following as conditional statements.

(a) p is a necessary condition for q

(b) r is a sufficient condition for s

(c) t is a necessary and sufficient condition for u

5. Come up with statements p and q such that...

(a) ... p is a sufficient condition for q .

(b) ... p is a necessary condition for q .

(c) ... p is a necessary and sufficient condition for q .

(d) ... p is neither a necessary nor sufficient condition for q .

6. Use symbols to write the logical form of each argument below. Then use a truth table to determine the validity of the argument. In your truth table, indicate which columns correspond to the premises and which correspond to the conclusion.

Argument 1:

If Lauren spells the word incorrectly, she loses the spelling bee.

Lauren spells the word correctly.

\therefore Lauren wins the spelling bee.


(problem 6, continued)

Argument 2:

A rock contains either gold or pyrite (fool's gold).
If the rock contains gold, it does not contain pyrite.
The rock contains pyrite.
 \therefore The rock does not contain gold.

7. Solve the following logic puzzle to find the treasure, assuming that each statement below is true.

If this house is next to a lake, then the treasure is not in the kitchen.
If the tree in the front yard is an elm, then the treasure is in the kitchen.
This house is next to a lake.
The tree in the front yard is an elm or the treasure is buried under the flagpole.
If the tree in the back yard is an oak, then the treasure is in the garage.

 this is
problem 37 on
page 62 of the
text

Did you use modus ponens or modus tollens in your reasoning above?

8. A set of premises and a conclusion are given below. Show how the conclusion follows logically from the premises.

$$\begin{aligned} & p \vee q \\ & q \rightarrow r \\ & p \wedge s \rightarrow t \\ & \sim r \\ & \sim q \rightarrow u \wedge s \\ \therefore & t \end{aligned}$$

BONUS: Identify each valid argument form (as listed in Table 2.3.1 on page 60 of the text) used in your reasoning above.