Math 234

Predicates and Quantified Statements — Day 5

Discuss the following problems with the people at your table.

1. Let $P(x)$ be the predicate "x is an ash tree" and let $Q(x)$ be the predicate "x is vulnerable to the emerald ash borer."

   D: set of all trees

   Consider the sentence:

   Every ash tree is vulnerable to the emerald ash borer.

   (a) Rewrite the sentence above as a universal conditional statement using the symbols $P(x)$ and $Q(x)$.

   $\forall\ x \in D$, if $P(x)$ then $Q(x)$.

   (b) What is the *negation* of your universal conditional statement in part (a)?

   $\exists\ x \in D$ such that $P(x)$ and $\sim Q(x)$

   There exists a tree that is an ash tree (and) but is not vulnerable.

   (c) What is the *contrapositive* of your universal conditional statement in part (a)?

   $\forall\ x \in D,\ \sim Q(x) \rightarrow \sim P(x)$.

   (d) What is the *converse* of your universal conditional statement in part (a)?

   $\forall\ x \in D$ such that $Q(x) \rightarrow P(x)$.

   (e) What is the *inverse* of your universal conditional statement in part (a)?

   $\forall\ x \in D,\ \sim P(x) \rightarrow \sim Q(x)$

2. Write the following statements in formal logic notation, being sure to specify all of the predicate symbols that you use. Then write the negation of each statement, both in English and using formal logic symbols.

   $x \in D$, D is set of polygons

   (a) If a polygon is a triangle, then its interior angles sum to 2π radians.

   $P(x)$: x is a triangle, $Q(x)$: int angles of x sum to 2π

   $\forall\ x \in D,\ P(x) \rightarrow Q(x)$

   neg: $\exists\ x \in D,\ P(x) \wedge \sim Q(x)$ → There is some polygon that is a triangle and whose angles don't sum to 2π rad.

   (b) If a polygon has interior angles that sum to 2π radians, then it is a triangle.

   $\forall\ x \in D,\ Q(x) \rightarrow P(x)$

   neg: $\exists\ x \in D,\ Q(x) \wedge \sim P(x)$

3. Determine whether each statement is true or false.

(a) Every hunk of iron $\underbrace{\text{that floats in water}}_{P(x)}$ $\underbrace{\text{is a hunk of gold}}_{Q(x)}$.

Domain: hunks of iron

Are you sure?

$$\forall x, \text{ if } P(x) \text{ then } Q(x)$$

VACUOUSLY TRUE

negations

(b) There is a hunk of iron $\underbrace{\text{that floats in water}}_{P(x)}$, which is $\underbrace{\text{not a hunk of gold}}_{\sim Q(x)}$.

$$\underbrace{\exists x \text{ such that } P(x)}_{\text{false}} \wedge \sim Q(x) \text{ — FALSE}$$

(c) Every number that is both even and odd is a multiple of 6.

Domain: $\mathbb{R}$

$P(x)$: x is both even and odd

$Q(x)$: x is a multiple of 6

VACUOUSLY TRUE

$$\forall x \in D, \ P(x) \to Q(x)$$

negations

(d) There is a number that is both even and odd, which is not a multiple of 6.

$$\underbrace{\exists x \in D \text{ such that } P(x)}_{\text{FALSE}} \wedge \sim Q(x) \text{ — FALSE}$$

4. Decide whether each statement is true or false. Then write the statement in formal logic symbols, specifying all predicate symbols that you use. Lastly, negate each statement.

(a) To earn an A in this class, it is necessary to be enrolled in this class. TRUE

For all students x, $\underbrace{\text{if } x \text{ is not enrolled}}_{P(x)}$, then $\underbrace{x \text{ cannot earn an A}}_{Q(x)}$.

$$\forall x, \ P(x) \to Q(x)$$

neg: There exists a student who is not enrolled but earns an A.

$$\exists x \text{ such that } P(x) \wedge \sim Q(x).$$

(b) To earn an A in this class, it is sufficient to be enrolled in this class.

5. Consider the following statement:

   There exists a student who has taken every course at St. Olaf College.

   (a) Express the statement above using logical symbols, defining each predicate symbol that you use.

   How many quantifiers do you need?

   (b) What is the negation of the statement above?

6. The *multiplicative inverse* of a real number x is a real number y such that $xy = 1$. Consider the following statement:

   Every real number has a multiplicative inverse.

   (a) Express the statement above using logical symbols, defining each predicate symbol that you use.

   (b) What is the negation of the statement above?

   (c) Which is true, the original statement or its negation?