

What virtues have you acquired as a result of doing mathematics?

Fibonacci Numbers

$$F_0 = 0, \quad F_1 = 1, \quad \text{and} \quad F_n = F_{n-1} + F_{n-2}$$

Last time: $F_{n+1} F_{n-1} - F_n^2 = (-1)^n$

SKETCH OF PROOF:

Matrix formula: $\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$ ← Proof by induction

Take determinants: $\det \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} = \det \left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \right)$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$F_{n+1} F_{n-1} - F_n^2 = (-1)^n$$

↖ Cassini's Identity

CONJECTURE: $F_{n-k} F_{n+k} - F_n^2 = F_k^2 (-1)^{?}$ — figure out what goes here