MATH 242 — 27 October 2025

How many primes are there? Infinitely many!

<u>Proof</u>: Suppose there are only finitely many primes:

$$p_1, p_2, p_3, p_4, \ldots, p_n.$$

biggest prime

Let $M = \underline{p_1 p_2 p_3 p_4 \cdots p_n} + 1.$

- Is M prime? No, M is bigger than any prime in our list.
- Is M composite? No, it's not divisible by any prime since <u>every</u> prime divides $M-1$.

CONTRADICTION! Therefore, there must be infinitely many primes.

SIEVE OF ERATOSTHENES

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	$=\sqrt{max}$
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~	
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~	
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~	
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~	
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~	
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~	
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~	
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~	$= max$

$n = p \cdot q$

If $n \leq 100$ and $n = pq$ where $p, q > 1$

then it must be that either $p \leq 10$ or $q \leq 10$.

So: To find primes up to max, it suffices to check for divisibility by factors up to $\sqrt{max}$.