Monopoly Project

Math 242

due Friday, April 27

Your assignment is to verify Tables 1, 2, 3, and 4 from the "Take a Walk on the Boardwalk" paper (`http://www.jstor.org/stable/2687519`).

How to include the "stay in Jail as long as you can" strategy

Start with the 42×42 rolling transition matrix. States 1 through 40 are the "normal" states, with state 11 the "just visiting" jail space. States 41 and 41 are Jail (Roll 1) and Jail (Roll 2).

The rolling probabilities of transitions for states 1 through 40 are the same as before. However, in this case make sure you include the zero probabilities of getting to States 41 and 42 from any of these States. The rolling probabilities of transitions for 41 and 42 take into account the intent to stay in for up to three rolls, unless doubles are rolled. This means:

- $41 \to 42$ and $42 \to 11$ with probability $5/6$ (no doubles).
- 41 and 42 transition to each of $13, 15, 17, 19, 21$, and 23 each with probability $1/36$.
- All other transition probabilities are zero.

Use column sums to verify that you have a legal transition matrix. Once you've done this you can now move to the Jail, Chance and Community Chest transitions.

The 42×42 jail transition matrix starts as the identity matrix. The only change is that State 31 ("Go to Jail") transitions to Jail (Roll 1) with probability 1. Use column sums to verify that you have a legal transition matrix.

Note that there are 3 states that cause a *Chance* card to be drawn, and 3 that cause a *Community Chest* card to be drawn. There are 16 cards of each type. *Chance* has ten cards that send you to a new state. *Community Chest* only has 2 cards that send you to a new state. For each, you only need one matrix. Start with the 42×42 identity matrix, and then modify the three columns associated to the Chance/Community Chest locations. Use column sums to verify that you have a legal transition matrix.

The final 42×42 transition matrix is the product of your four matrices above. The order of multiplication is important. Use column sums to verify that you have a legal transition matrix.

How to include three doubles in a row

The idea is that from any of the regular (not extra jail) positions, you have $1/216$ probability of having rolled three doubles in a row (this is not exactly true, but it's very close, and prevents this step from being even more complicated). The transition matrix is therefore a matrix that says for each regular position you have a $215/216$ probability of staying put and $1/216$ probability of transitioning to Jail (Roll 1)
You can incorporate this by creating another 42×42 matrix. For i in $\{1, 2, \ldots, 40\}$, set entry (i, i) to $215/216$, and set entry $(41, i)$ to $1/216$. Think about how to incorporate this into the overall transition matrix.

Analysis of color groups

As a last step, once you have the steady state probabilities, group the states together by color and perform some additional analysis.

- What are the steady state probabilities for each color group? (Just add up the probabilities for all the properties in the group).
- Assuming the property is fully developed, how much does it earn per roll? (Multiply the total rent times the probability).
- If you know how much a property earns per roll, how many rolls does it take to "break even" for the property? (Divide total development cost by earnings per roll).

Data for the Monopoly properties is in the file MonopolyData.csv. In particular, the *Total* column gives the cost to fully develop a property, and the *Rent* column tells you how much the fully-developed property earns when someone lands on it. You can read this file into R via:

```
monopolyData <- read.csv("MonopolyData.csv")
```

Your Report

Turn in a HTML file knit using R Markdown. Make sure that you clearly explain your work, and include the items mentioned in the grading rubric below. Do not include superfluous output (e.g., you don't need to print all the entries in a 40×40 matrix). Your goal should be to communicate your work to another person (e.g., another student at your level who is not in this course).

Grading Rubric

Your notebook will be graded on a scale of 0 to 4, according to the following rubric.

4. Problems and goals are clearly stated, including relevant definitions or parameters. Computations are complete; code runs and is clearly explained. Conclusions are clearly stated and backed up by sufficient computational evidence. Limitations of the methodology, extensions for future work, and/or conjectures are discussed. Notebook is well-formatted and easy to read.

3. Problems and goals are stated well, though relevant definitions or parameters may be missing. Computations are mostly complete; code runs, but explanation is weak. Conclusions are unclear or not well justified. Insufficient discussion of limitations, extensions, and/or conjectures.

2. Statement of problem or goal is unclear. Computations are incomplete; explanation is ambiguous. Code may produce errors when run. Conclusions are possibly correct, but not justified. Little or no discussion of limitations, extensions, and/or conjectures. Notebook is difficult to read.

1. Serious misunderstanding of the problem or goal. Computation is inadequate for the task at hand. Work is not clearly explained. No discussion of limitations, extensions, and/or conjectures. Notebook is difficult to read.

0. Notebook is not turned in.