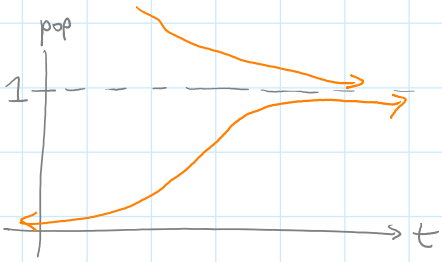
LOGISTIC MAP



Population growth models (diffential equation)

$$\frac{df}{dt} = r f(1-f)$$

models population growth with a carrying capacity

r = growth rate, $f(t)$ = population as a proportion of carrying capacity at time t

Discrete model: $$x_{n+1} = r\, x_n (1-x_n)$$

x_n = relative population size at time step n = $\frac{\text{current size}}{\text{carrying capacity}}$

r = growth rate constant

Investigation: Choose $0 < x_0 < 1$ and $0 < r < 4$

Iterate to compute $x_1, x_2, x_3, x_4, \ldots$ What happens?

Observations:

$r < 1$ — convergence to zero

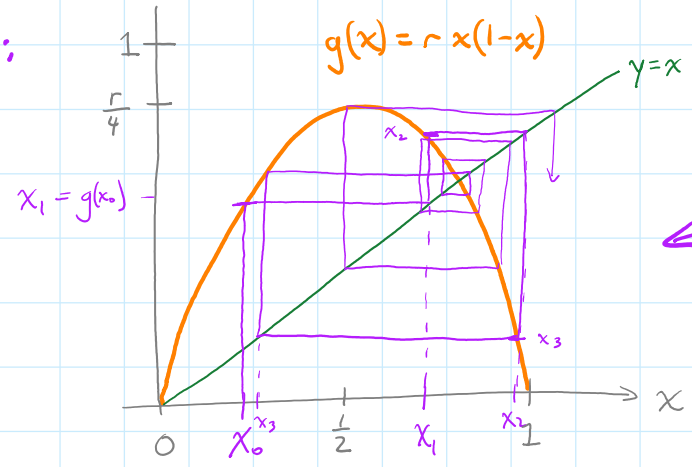
$1 < r < 2.88$? — convergence to some value

$3 < r < 3.3$? — alternates between two values

$r = 3.5$ — cycle of 4 values

$r = 3.8$ — crazy!

Visualization:



← cobweb plot

Bifurcation Diagram: