UNBOUNDED FUNCTION: $f(x)$ is unbounded if, for any (big) number M, there exists some x such that $f(x) > M$.

For the Mean-Median project, $f(\vec{x})$ is the number of steps, starting with sequence $\vec{x}$, until the iterative process stabilizes (becomes constant).

PRIME NUMBERS: 2, 3, 5, 7, 11, 13, 17, 19, ...

A positive integer p is prime if it has only two factors: 1 and p.

How many primes are there? Infinitely many!

Why? Assume there are finitely many primes:

$$2, 3, 5, 7, \ldots, p.$$

Then let $n = (2 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot p) + 1$.

Now n is not divisible by 2, or 3, or 5, etc.

In fact, n is not a multiple of any prime, so n must be prime.

This is a contradiction, since n is not in our list of all primes.

Thus, there are infinitely many primes. ∎

CURRENT RECORDS

- Largest known prime: $\underbrace{2^{82,589,933} - 1}_{\text{Mersenne prime}}$, which has 24,862,048 digits. (Dec. 2018)

- Arbitrary numbers up to more than 15,000 digits can be proven prime.
- RSA-250, a 250-digit number with exactly 2 prime factors was factored in Feb. 2020.
  This took roughly 2700 "core-years" of computing time.
- Arbitrary numbers of more than 200 digits can be factored.

Question: How would you determine if a positive integer is prime?

want: function `isPrime[n_]`:

argument: n is a positive integer

return: True if n is prime, False otherwise

Think and sketch an algorithm on paper before writing code.

Useful Functions:

`Mod[n, k]` returns the remainder when `n` is divided by `k`

`Divisible[n, k]` returns `True` if `n` is divisible by `k`, and `False` otherwise.