SIEVE OF ERATOSTHENES

LITERALIST IMPLENTATION

nums = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Loop: i from 1 to $\sqrt{n}$

  Loop: j from $i+1$ to end of the list

    If nums[[i]] divides nums[[j]], then delete nums[[j]]

FAST IMPLEMENTATION

nums = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

EXAMPLE CODE: $\underset{\text{start index}}{\text{nums[[2}}$;; $\underset{\text{stop index}}{n}$;; $\underset{\text{step size}}{2}$]] = 0

OBSERVATION: When we remove multiples of k, it suffices to start with k^2. Every smaller multiple of k has already been removed.

IMPLEMENTATION: nums = Range[n]

loop: k from 2 to $\sqrt{n}$

  if nums[[k]] $\neq 0$, then

    nums[[k^2 ;; n ;; k]] = 0

select all nonzero elements from nums

SIEVE OF SUNDARAM

ALGORITHM:

1. Start with a positive integer n.
2. Let list 1 contain all integers of the form $i+j+2ij$, where i and j are integers, $1 \le i \le j$, and $i+j+2ij \le n$.

   EXAMPLES: if $i=1$, $j=1$, then $1+1+2(1)(1) = 4 \in$ list 1

   3 is not in list 1, 5 is not in list 1
3. Let list 2 contain all integers $1, 2, \dots, n$ that are <u>not</u> in list 1.
4. For each number in list 2, double it and add 1.

   This gives a list of all odd primes up to $2n+1$.

Why does this work?

- The numbers resulting from the algorithm are odd integers m, with $3 \le m \le 2n+1$.
- Excluded numbers are of the form $q = 2(i+j+2ij)+1$
- Excluded numbers are precisely those with nontrivial odd factors.

$$q = \underbrace{4ij + 2i}_{2i(2j+1)} + \underbrace{2j + 1}_{+1(2j+1)}$$

$$q = (2i+1)(2j+1)$$

- Non-excluded numbers are therefore odd primes.