15 March 2024

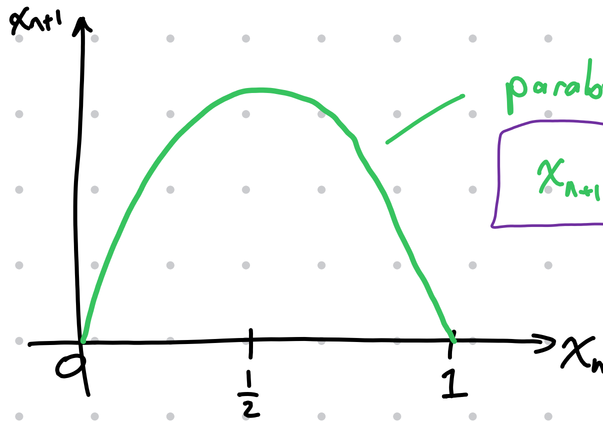
SCENARIO: Define a sequence of relative population sizes between 0 and 1.

$$x_0, x_1, x_2, x_3, x_4, \ldots$$

Idea: If x_n is close to zero, then so is x_{n+1}.
If x_n is close to one, then x_{n+1} is close to zero.
If x_n is close to $\frac{1}{2}$, then x_{n+1} is big.

Diff. Eq: logistic equation

$$\frac{df}{dx} = f(x)\big(1-f(x)\big)$$



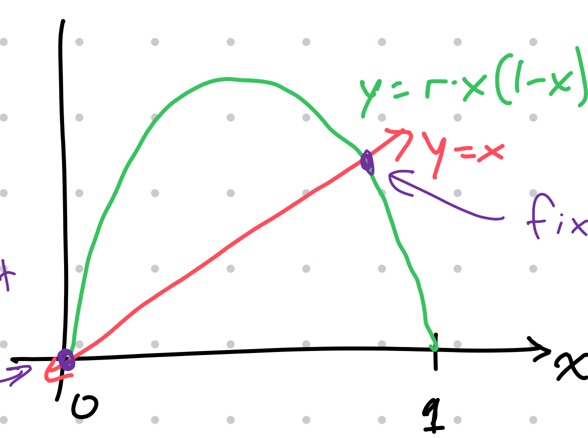
parabola: LOGISTIC MAP EQ

$$\boxed{x_{n+1} = r \cdot x_n (1-x_n)}$$

r is some constant "growth rate"

FIXED POINT: a value x^* such that

$$\underbrace{x^*} = \underbrace{r \cdot x^*(1-x^*)}$$



fixed point $x^* = 0$

fixed point: $\cancel{x^*} = r\cancel{x^*}(1-x^*)$

$$1 = r(1-x^*)$$

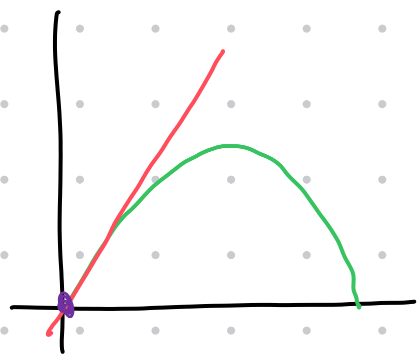
$$1 = r - rx^*$$

$$rx^* = r - 1$$

$$x^* = \frac{r-1}{r}$$

If $r=2$, then $x^* = \frac{2-1}{2} = \frac{1}{2}$

If $r=1$:



SUMMARY:

- If $0 \leq r \leq 1$, then x_n converges to 0.
- If $1 < r < 3$, then x_n converges to the other fixed point $x^* = \frac{r-1}{r}$.
- If r is somewhat larger than 3, we observe oscillation between 2 values.
- For larger r, oscillation between 4 values,

At $r=1$, we observe a <u>bifurcation</u>: a qualitative change in the behavior of the system.

COBWEB PLOT