

20 March 2024

The Logistic Map

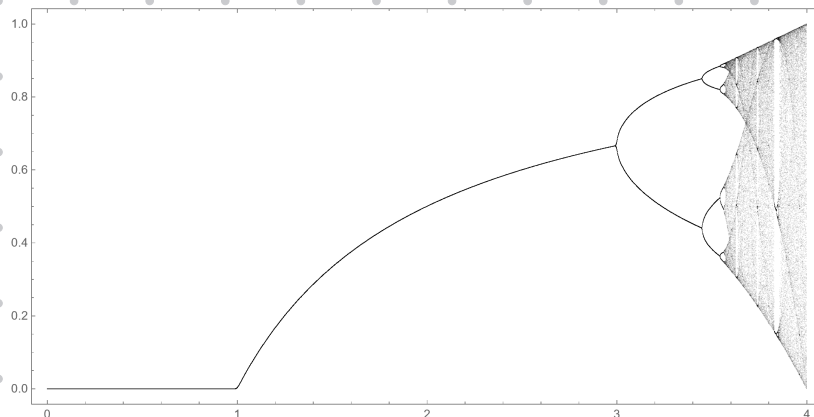
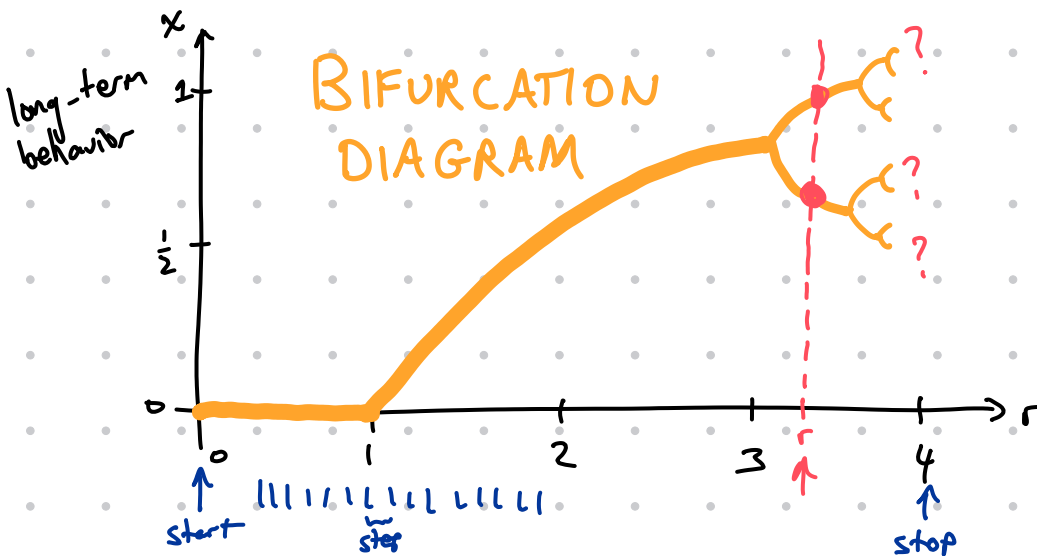
$$f_r(x) = r \cdot x(1-x)$$

$$0 \leq r \leq 4$$

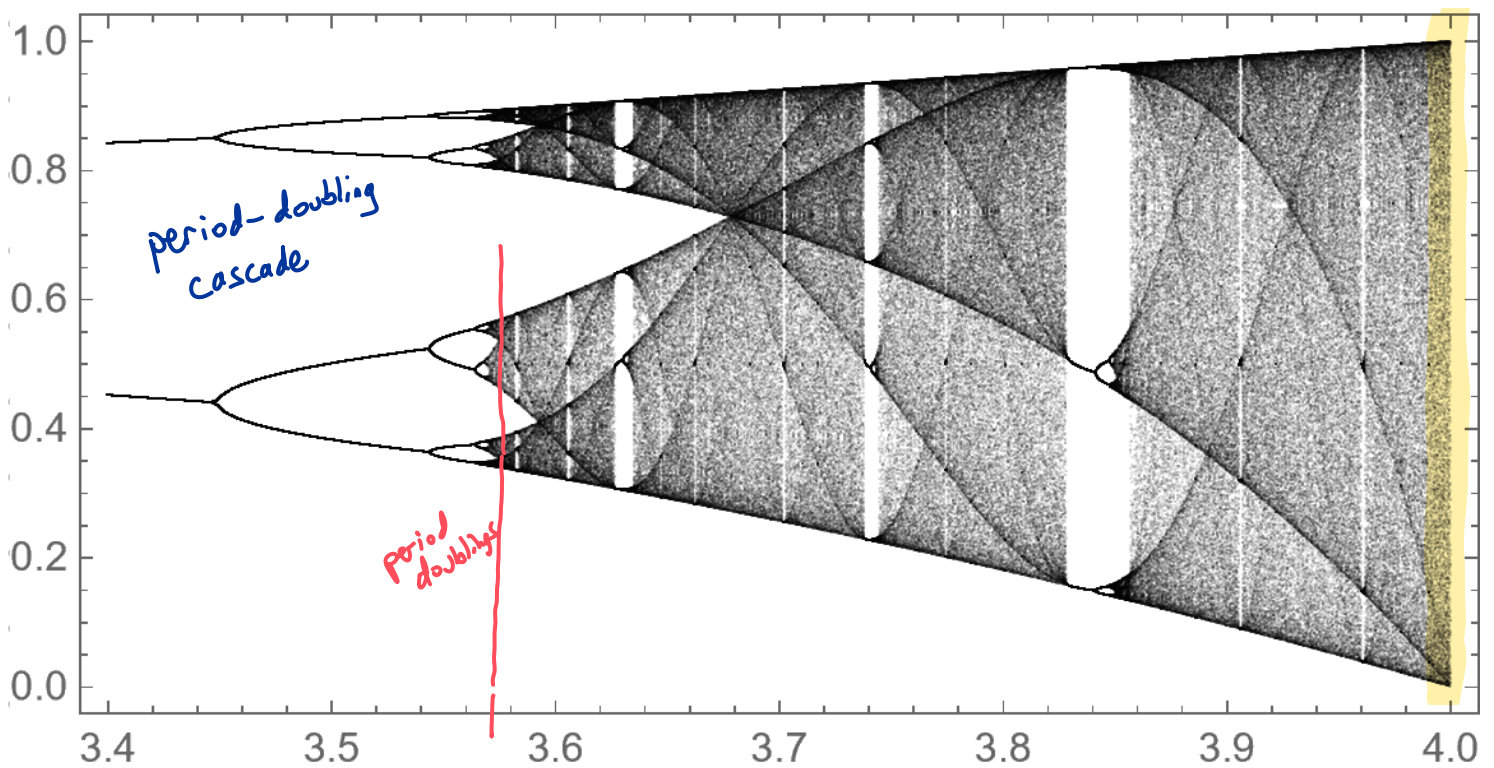
$$0 \leq x \leq 1$$

Recall:

- If $0 \leq r \leq 1$, then the trajectories converge to 0.
- If $1 < r \leq 3$, the trajectories converge to $x = \frac{r-1}{r}$.
- If $3 < r < 3.449$, the trajectories oscillate between two values.
- As r increases beyond 3.45, we observe a sequence of period-doubling bifurcations.



Question: Can you find evidence of k -cycles for integers k that are not powers of 2?



For many values of r , $3.57 < r \leq 4$, the trajectory never enters a repeating cycle.

For $r=4$:

- trajectories visit every small interval in $[0,1]$
- trajectories exhibit sensitive dependence on initial conditions

chaos