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How many primes are there?

Assume there are finitely many primes, namely

$$p_1, p_2, p_3, \ldots, p_k$$

Mutiply all the primes together and add 1:

$$N = p_1 \cdot p_2 \cdot p_3 \cdots p_k + 1$$

N cannot be prime since it is larger than all of the primes in our list.

Then N must have some prime factor.

- p_1 cannot divide N since N is one more than a multiple of p_1
- p_2 cannot divide N, by similar reasoning
- similarly, none of the primes in our list divide N.

Contradiction!

Thus, there must not be finitely many primes.

THEOREM: There are infinitely many prime numbers.

If $n = a \cdot b$, where a and b are pos. ints, not 1 and n,

then either a or b is not greater than $\sqrt{n}$.

Why? If $a > \sqrt{n}$ and $b > \sqrt{n}$,

then $a \cdot b > \sqrt{n} \cdot \sqrt{n} = n$.

$24 = 4 \cdot 6$

$4 < \sqrt{24}$

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~ $= n$

Sieve of Eratosthenes

- start with a list $2, 3, 4, 5, \ldots, n$
- REPEAT: smallest remaining number is prime
  cross off all multiples of that number
- STOP when you reach $\sqrt{n}$.