Iterated Functions Project

Math 242, spring 2024

Due: Monday, April 8

(Following the due date and initial grading, there will be an opportunity to revise and resubmit for a higher grade.)

In this project, you will explore sequences obtained by iterating the following variant of the Collatz function:

$$\mathrm{col}_5(n) = \begin{cases} 3n+5 & \text{if } n \text{ is odd,} \\ \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$$

Your task is to understand as much as you can about the trajectories of $\mathrm{col}_5(n)$. Specifically, consider the following questions:

1. What cycle(s) do you observe when in the trajectories of $\mathrm{col}_5(n)$ for positive integers n?
2. If there is more than one cycle, how often does each cycle occur? Are these proportions stable as n increases?
3. What can you say about the heights of the trajectories? How about the stopping times?
4. How do your observations about $\mathrm{col}_5(n)$ compare or contrast with your observations about $\mathrm{col}(n)$?
5. For a score of Excellent, investigate at least two other questions of your choosing.

For projects in Math 242, *communication* is as important as *computation*. You should turn in a well-organized notebook that clearly explains, using sentences and paragraphs, what you computed and what conclusions you can draw.

This project will be graded on the EMRN scale, as described in the syllabus. To receive a grade of *Meets Expectations*, your notebook should exhibit the following characteristics:

- You demonstrate computational exploration of the $\mathrm{col}_5(n)$ trajectories.
- Your observations allow you to answer the questions above, and you make conjectures as appropriate.
- Your reasoning is explained using sentences, and your notebook is well-formatted and easy to read.
- No significant gaps or errors are present.

To receive a grade of *Excellent*, your notebook should further exhibit the following:

- You have investigated at least two of your own questions (item 5 above). These investigations must go beyond minor modifications of code from class.
- Computational methodology demonstrates mastery of the computational techniques that we have studied in this course.
- Mathematica code is of high quality, demonstrating skillful use of programming constructs (e.g., variables, lists, functions, modules).
- Exposition is clear and precise, thoroughly explaining your methodology and reasoning. Any assumptions necessary for the estimates are reasonable and clearly stated.
- The work may extend beyond the project requirements in a creative or insightful direction.