FIBONACCI SEQUENCE

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

$F_0 \quad F_1 \quad F_2 \quad F_3 \qquad\qquad\qquad F_{10}$

Recurrence: $F_n = F_{n-1} + F_{n-2}$

Conjecture: The Fibonacci numbers satisfy

$$\lim_{n \to \infty} \frac{F_n}{F_{n-1}} = \frac{1 + \sqrt{5}}{2} = \phi.$$

Observations

$$\lim_{n \to \infty} \frac{F_n}{F_{n-2}} = 1 + \phi = \phi^2$$

$$\lim_{n \to \infty} \frac{F_n}{F_{n-3}} = \phi^3$$

Could it be that $\lim_{n \to \infty} \dfrac{F_n}{F_{n-k}} = \phi^k = \left(\dfrac{1 + \sqrt{5}}{2}\right)^k$?

Another conjecture?

Compare: $F_{n-1} F_{n+1}$ with F_n^2

mathematical identity
↓

Conjecture: The Fibonacci numbers satisfy $F_{n-1} F_{n+1} - F_n^2 = (-1)^n$ for all positive indexes n.