MATH 242 — 13 March 2026

Francis Su writes:

"Even wrong ideas soften the soil in which good ideas can grow"

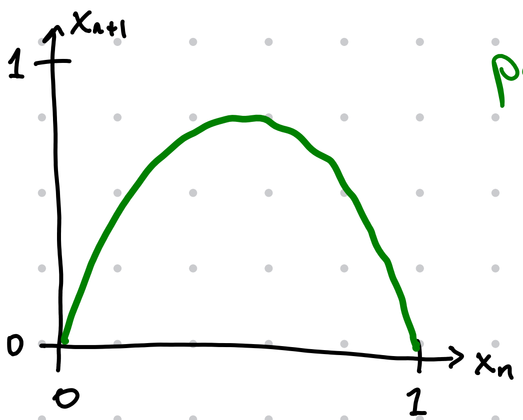
How have you seen this in your own life or in your own mathematical experience?

SCENARIO: Sequence of relative population sizes between 0 and 1: $x_0, x_1, x_2, x_3, \ldots$

Idea: If x_n is close to 0, then x_{n+1} is also small.

If x_n is close to 1, then x_{n+1} is close to 0.

If x_n is close to $\frac{1}{2}$, then x_{n+1} is big.



parabola:

$$\underbrace{x_{n+1} = \boxed{r \cdot x_n (1 - x_n)}}_{\text{Logistic Map equation}}$$

define $f_r(x) = r \cdot x(1-x)$

$0 \le r \le 4$

Choose x_0 and compute $x_1, x_2, x_3, \ldots$

In diff. eq., the logistic equation is

$$\frac{df}{dx} = r \cdot f(x)\big(1 - f(x)\big)$$

Why does population sometimes stabilize?

If: $x = f_r(x)$

$$x^* = r \cdot x^*(1-x^*)$$

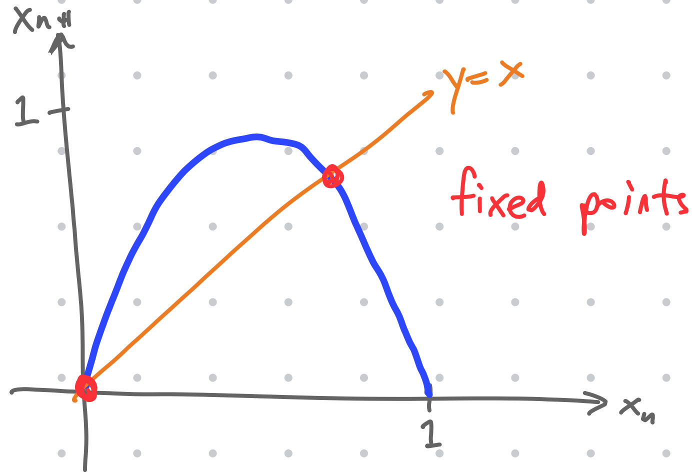
Solve for x^*:

$\boxed{x^* = 0}$ or $1 = r(1-x^*)$

$1 = r - x^* r$

$x^* r = r - 1$

$$\boxed{x^* = \frac{r-1}{r}}$$



Check: If $r=2$, $x^* = \frac{2-1}{2} = \frac{1}{2}$

If $r=1.5$, $x^* = \frac{1.5-1}{1.5} = \frac{0.5}{1.5} = \frac{1}{3}$

Cobweb Plot: