MATH 242 — 16 March 2026

THE LOGISTIC MAP

$$f_r(x) = r \cdot x(1-x)$$

parameter: $0 \le r \le 4$

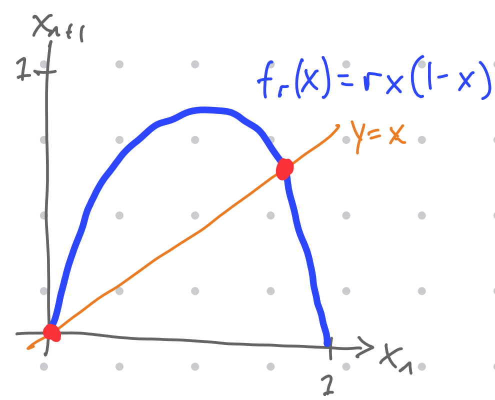
variable: $0 \le x \le 1$

Observations:

If $0 \le r < 1$, then the population size x_n converges to 0.

bifurcation

If $1 \le r < 3$, then the population size converges to a nonzero fixed point $x^* = \frac{r-1}{r}$.



bifurcation

What happens for $r > 3$?

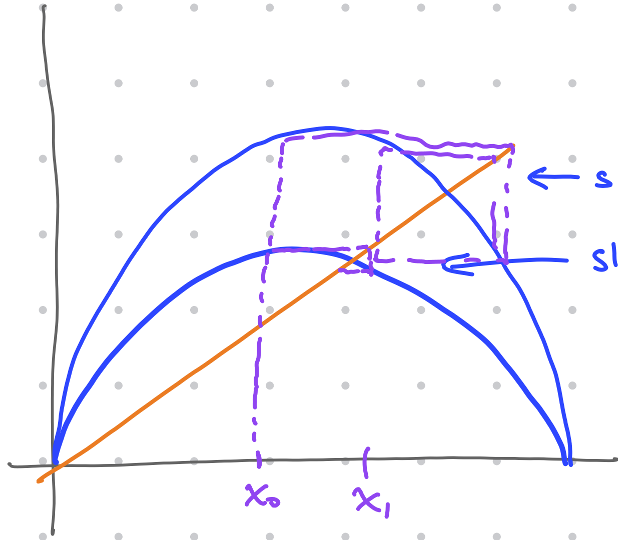
If $3 \le r < \underline{3.4?}$, then the population size enters oscillation between two values

bifurcation

If $3.4 < r < \underline{\ ?\ }$, then the population size enters oscillation among 4 values?

If $3.\_\_ < r < 3.\_\_$ then ... oscillating among 8 values.

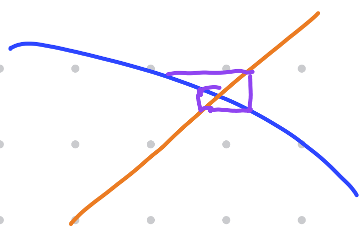
BIFURCATION: A qualitative change in the long-term behavior of a system.



← slope is big — $|f'(x^*)| > 1$

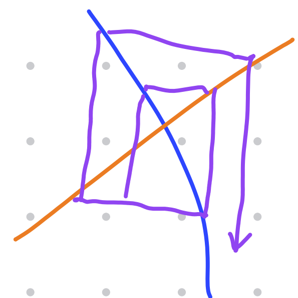
← slope is small at the fixed point — $|f'(x^*)| < 1$

$$x = f_r(x)$$



$|\text{slope}| < 1$

attracting fixed point (stable)



$|\text{slope}| > 1$

repelling fixed point (unstable)

If trajectory oscillates between <u>two</u> values, this means that $f_r(f_r(x)) = x$ has some fixed points

Let $f_r^{(2)}(x) = f_r(f_r(x))$.