An Art Gallery Theorem

MATH 261 Computational Geometry

Theorem: To cover a polygon with n vertices, $\lfloor n/3 \rfloor$ guards are necessary for some polygons and sufficient for all of them.

Proof that $\lfloor n/3 \rfloor$ guards are necessary:
Describe a family of polygons that shows this.

Proof that $\lfloor n/3 \rfloor$ guards are sufficient:
Let P be a polygon with n vertices. We know that P can be triangulated.
Show that each vertex of P can be assigned one of three colors so that any pair of vertices connected by an edge of P or a diagonal of the triangulation have different colors.

To complete the proof, explain how the least-used vertex color gives a placement of at most $\lfloor n/3 \rfloor$ guards that cover the polygon.

Choose one of the following problems to think about with your group.

Orthogonal polygons: How could you show that $\lfloor n/4 \rfloor$ guards are necessary for some and sufficient for all orthogonal polygons?

Algorithms: How could you program a computer to find a placement of at most $\lfloor n/3 \rfloor$ guards that cover a given polygon? Suppose that the polygon is specified by a list of (x, y) coordinates of its vertices, in order around the polygon.