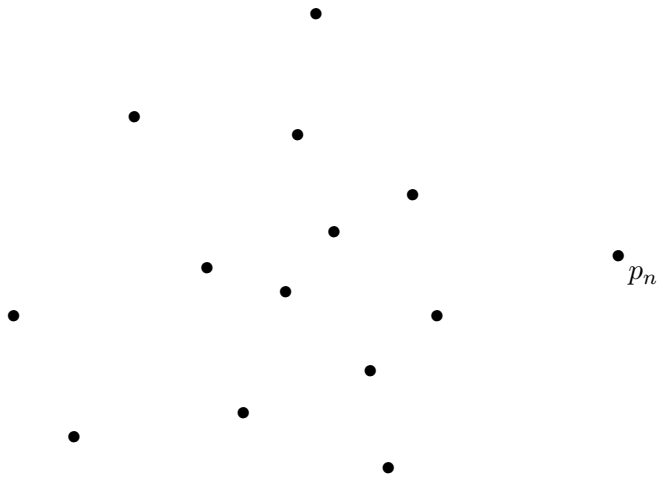
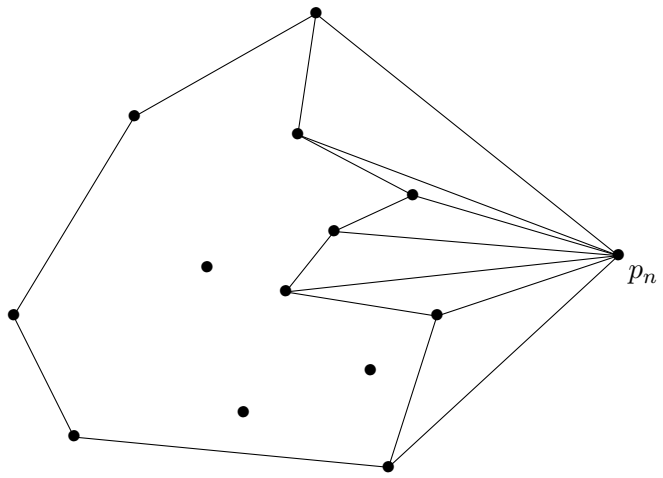
Triangulations and Edge Flips

MATH 261 Computational Geometry

1. Consider the following set of points S. Label the points $p_1, p_2, \ldots, p_n$ from left to right. If the incremental algorithm is used to triangulate S, which triangles are incident to p_n? Draw all such triangles below.



2. Suppose a triangulation of S (same S as above) includes the edges shown below. Find a sequence of edge flips that transform these edges into the edges you drew above. That is, you want to transform the edges such that the triangles incident to p_n are exactly those produced by the incremental algorithm on S.



3. Generalize your observations from #1 and #2. Let S be *any* set of n points in the plane, and let p_n be the rightmost point of S. Given *any* triangulation of S, can you always find a sequence of edge flips that result in the triangles incident to p_n being exactly those produced by the incremental algorithm on S? If so, find an algorithm that achieves the edge flips. If not, give a counterexample.

4. In your algorithm for #3, what is the largest number of edge flips that might be required? (Your answer should depend on n).

5. What is the largest number of edge flips that might be required to transform some triangulation of S into the triangulation produced by the incremental algorithm? What does this imply about the diameter of the flip graph of S? (The *diameter* of the flip graph is the length of the longest path between any two nodes in the graph.)