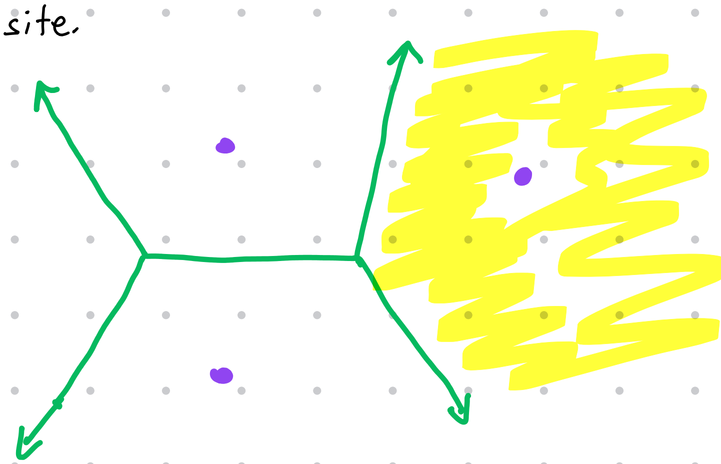
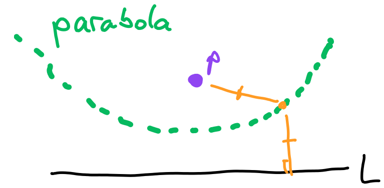
17 January 2025

VORONOI DIAGRAM

Start with "sites" in the plane

Partition the plane into regions, each of which consists of those points closer to a particular site than to any other site.





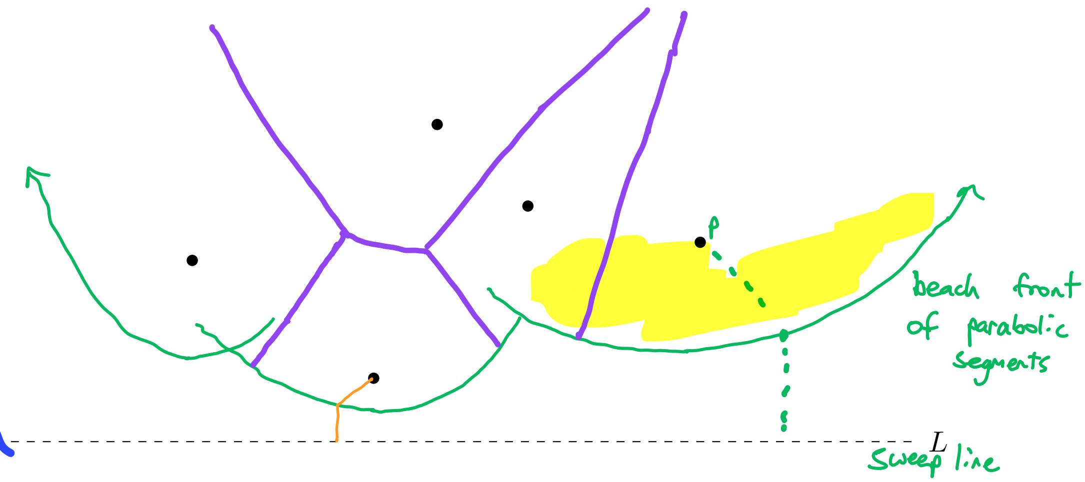
set of points equidistant from a focus point and a directrix line

Another Voronoi Algorithm

MATH 261 Computational Geometry

Suppose you have only partial knowledge of the sites from which a Voronoi diagram is to be constructed. Specifically, you can see only those sites to above a given horizontal line L, as shown here:

FORTUNE'S ALGORITHM



? ? ?

1. How much of the Voronoi diagram can you determine from this information?

2. How does this diagram suggest an algorithm for computing the Voronoi diagram?

FORTUNE'S ALGORITHM

Maintain a list of events, ordered by y-coordinate.
↳ priority queue

Process two types of events.

- **site events:** When the sweep line crosses a new site, add a new parabolic arc to the beach front
- **Circle event:** When a parabolic segment disappears, we encounter a Voronoi vertex } correspond to Voronoi vertices

As we process the events, ordered by y-coordinate, we record Voronoi vertices and the edges that connect them.

COMPLEXITY: If there are n sites, then the algorithm must process:

- n site events
- at most $2n-5$ circle events

} $O(n)$ events

maintaining the priority queue takes $O(\log(n))$ time per event

$O(n \cdot \log n)$

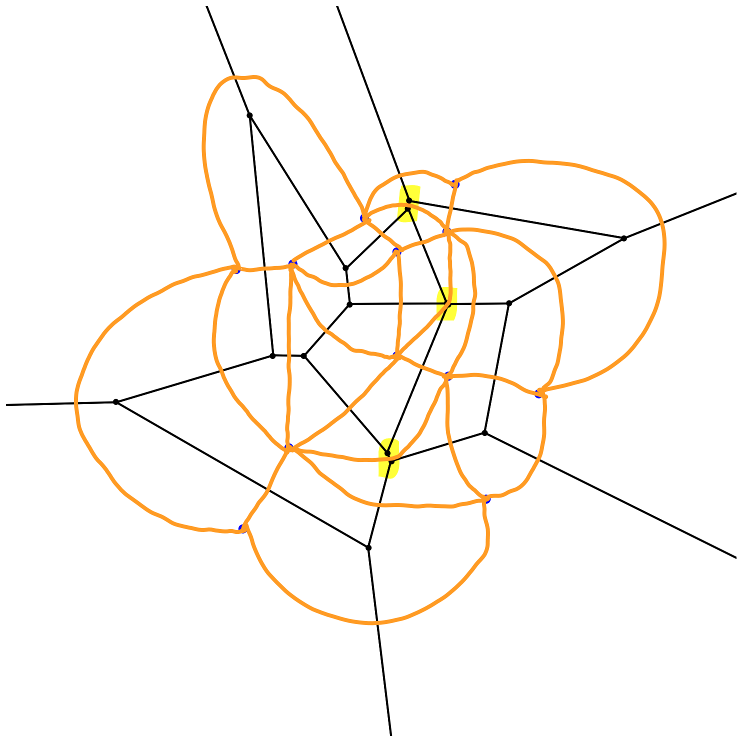
Sorting the sites takes $O(n \cdot \log(n))$ time).

total complexity $O(n \cdot \log(n))$

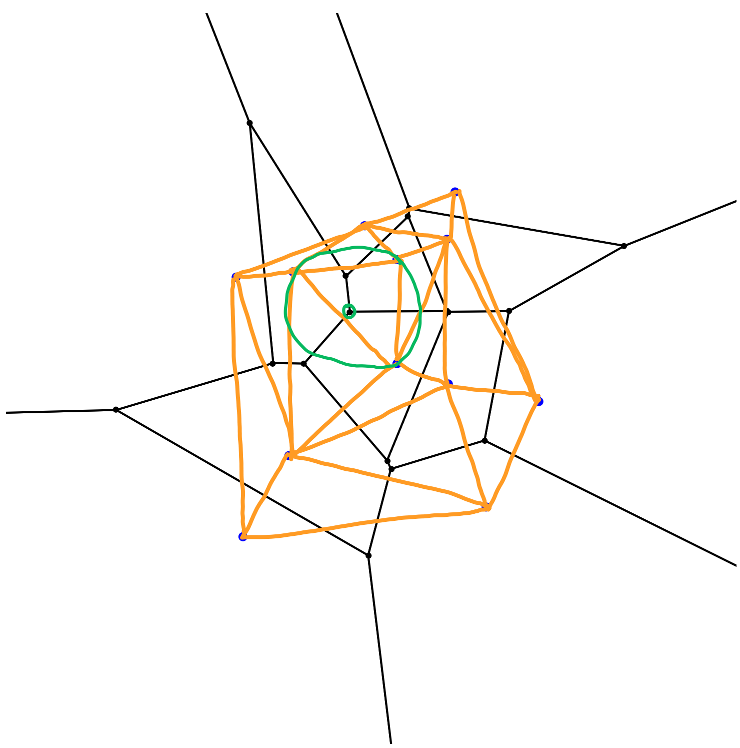
Voronoi Diagrams

MATH 261 Computational Geometry

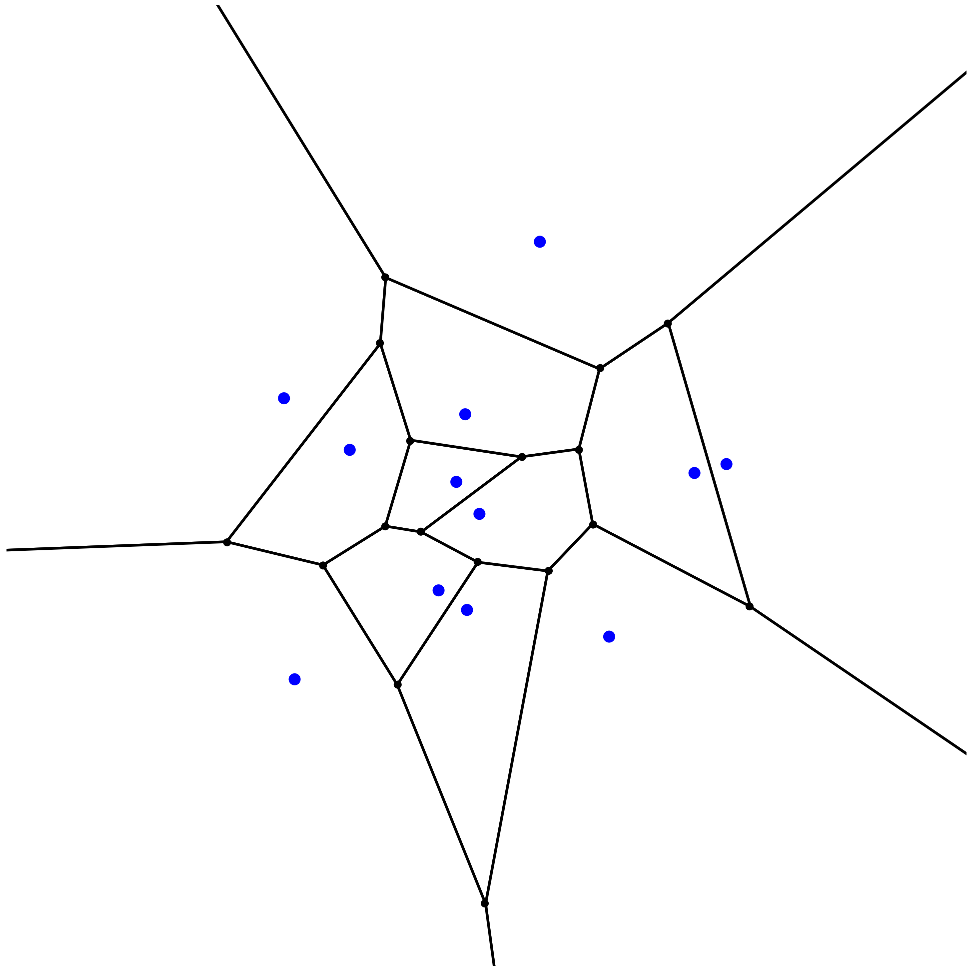
1. Draw the dual graph of the following Voronoi diagram.



2. Now draw the dual graph again, with straight-line edges.



3. Draw the straight-line dual graph of the following Voronoi diagram.



4. Make a conjecture about the dual graph of a Voronoi diagram. Then try to prove your conjecture.

The dual graph of a Voronoi diagram
is a Delaunay triangulation

VORONOI DIAGRAMS AND DELAUNAY TRIANGULATIONS ARE DUALS!

THEOREM: If S is a point set with no four points cocircular, then the dual graph of $Vor(S)$ is the Delaunay triangulation of S.

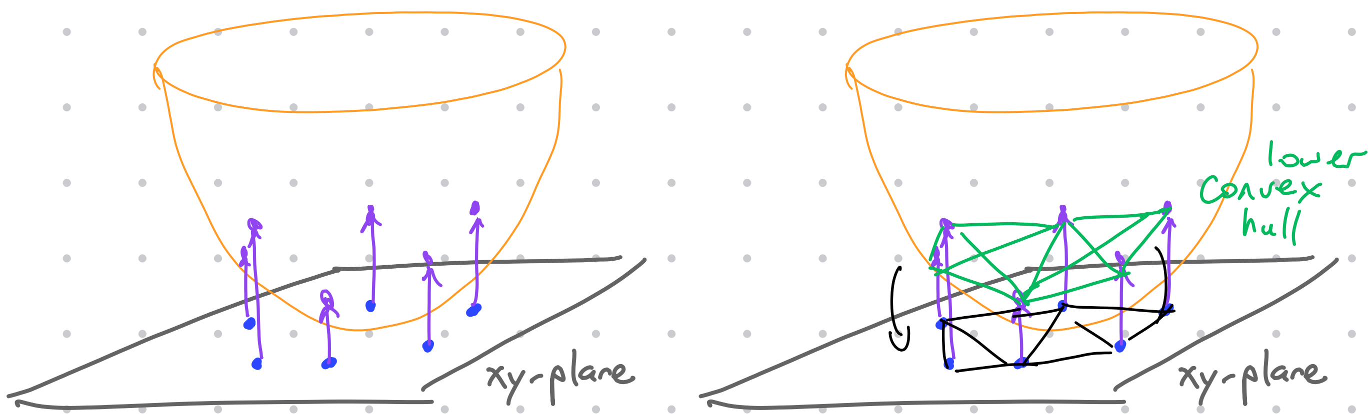
proof: A point v is a Voronoi vertex if and only if there is a circle C centered at v with three sites on its boundary and none in its interior.

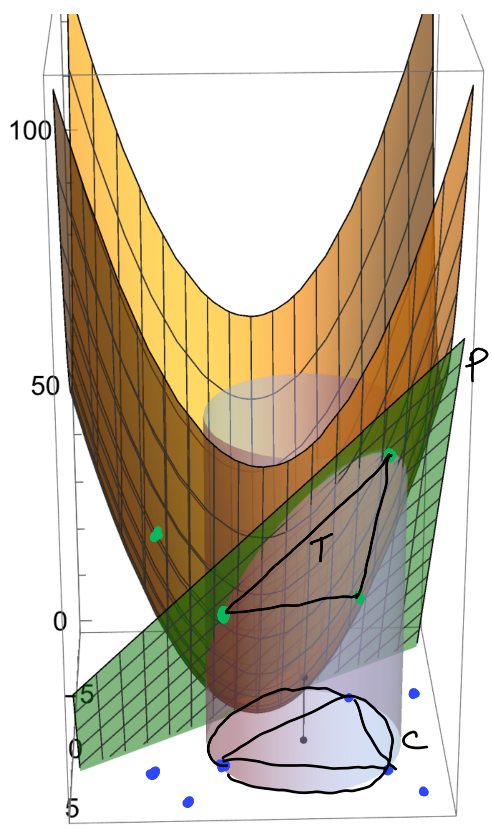
Surrouning v are 3 Voronoi regions which are all adjacent, producing a triangle in the dual graph. This triangle has as its three vertices the 3 sites on the circle C.

No other sites are in the circle, so the circle C satisfies the empty circle property of the Delaunay triangulation.

The same holds for all triangles in the dual graph, so the dual graph is the Delaunay triangulation of S.

CONVEX HULL, VORONOI, AND DELAUNAY





Choose a triangle T of the lower hull.
Let P be the plane containing T.

The intersection of plane P with the paraboloid is an ellipse, which projects down to circle C in the xy-plane

Since T is a face of the lower hull, all other sites on the paraboloid must be above plane P, and thus project outside of circle C in the xy-plane.

Thus, circle C contains 3 sites on its boundary and none in its interior.

So C satisfies the empty circle property of the Delaunay triangulation.