23 January 2025

POLYHEDRA

NOT-QUITE-PRECISE DEFINITION:

A polyhedron is a bounded region of space whose boundary is composed of a finite set of flat polygonal faces, any two of which are either disjoint or meet at edges and vertices.

A polyhedron P is CONVEX if, for all points x and y in P, the line segment $\overline{xy}$ is contained in P.

A convex polyhedron is REGULAR if all of its faces are congruent regular polygons, and the same number of faces meet at each vertex.

Regular Polyhedra

MATH 261 Computational Geometry

1. What regular polyhedra are possible with each of the following types of faces?

Hint: How many of each face could possibly meet at each vertex?

regular polygons

(a) Triangles (equilateral)
(b) Squares
(c) Pentagons
(d) n-gons for $n \geq 6$

tetrahedron
4 faces

3 triangles per vertex

octahedron
8 faces

4 triangles per vertex

PLATONIC SOLIDS

icosahedron
20 faces

5 triangles per vertex

~~6~~ triangles per vertex — no room to fold

SQUARES: 3 squares per vertex makes a cube

6 faces total

~~4~~ squares per vertex

PENTAGONS: 3 pentagons per vertex

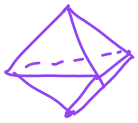
dodecahedron
12 faces

~~Hexagons?~~

2. Suppose you relax the definition of regular polyhedra to allow *different* numbers of faces to meet at each vertex. What new polyhedra will now be possible?

Combine 2 tetrahedra

6 triangular faces



triangular dipyramid

pentagonal dipyramid

3. Suppose you relax the definition of regular polyhedra to allow nonconvex polyhedra. What polyhedra will result?

glue together several congruent regular polyhedra along faces

stellated polyhedra: make each face into a pyramid

SEMI-REGULAR POLYHEDRA

Convex polyhedra with more than one type of regular polygonal faces, all corners are identical (same number of edges meet at same angles at each vertex).

- 13 Archimedean solids
- Infinite families: prisms
  antiprisms

Semi-Regular Polyhedra

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4. What semi-regular polyhedra can be obtained by truncating the corners of...

(a) a tetrahedron?

(b) a cube?

(c) an octahedron?

(d) a dodecahedron?

(e) an icosahedron?

Does it matter *how much* you truncate the corners in each case?

5. What prisms are semi-regular polyhedra?

6. Can you think of any other semi-regular polyhedra?