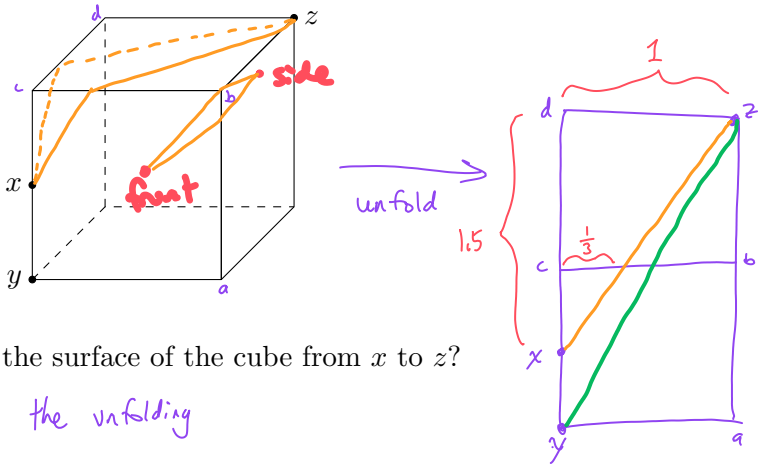
29 January 2025

Shortest Paths on Polyhedra

MATH 261 Computational Geometry

1. Consider a unit cube:

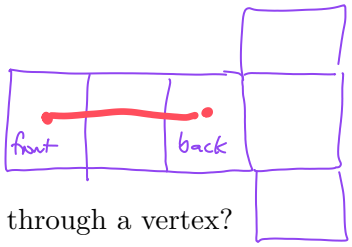


(a) What is the shortest path on the surface of the cube from x to z?

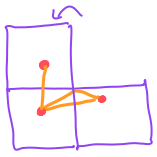
Straight line in the unfolding

(b) What is the shortest path on the surface of the cube from y to z?

(c) Does the shortest path between any two points ever touch the interior of 4 or more faces of the cube?



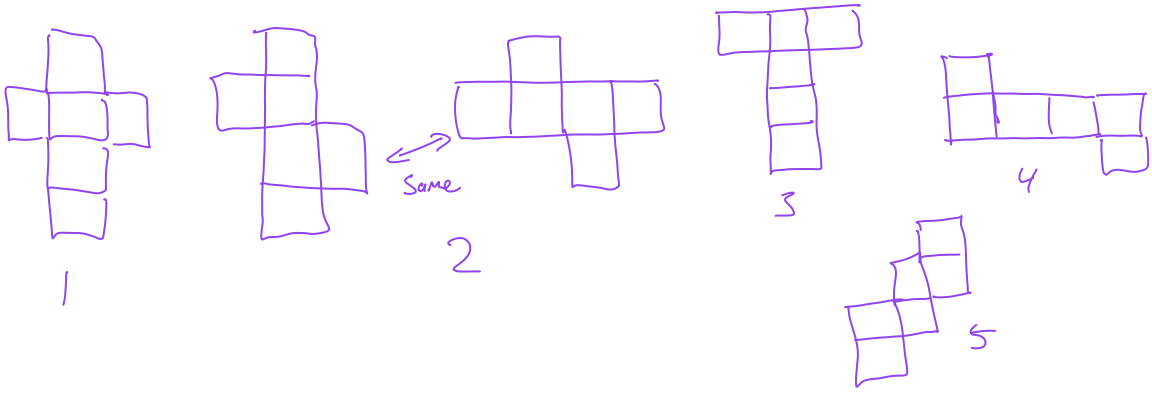
(d) Does the shortest path ever go through a vertex?



No, unless one endpoint is a vertex

Same

2. How many ways are there to unfold a cube? 11



If you cut some edges of a polyhedron so that it unfolds flat, you have created a **Net** for your polyhedron.

Does every polyhedron have a net?

NET: cut along edges

GENERAL NET: cut along edges or through faces

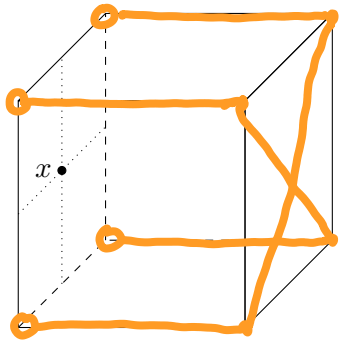
	polyhedra	
	convex	not convex
edge unfolding (net)	open question	NO
general unfolding	YES	open question

CUT LOCUS: $C(x)$ is the closure of the set of all points y to which there is more than one shortest path to x.

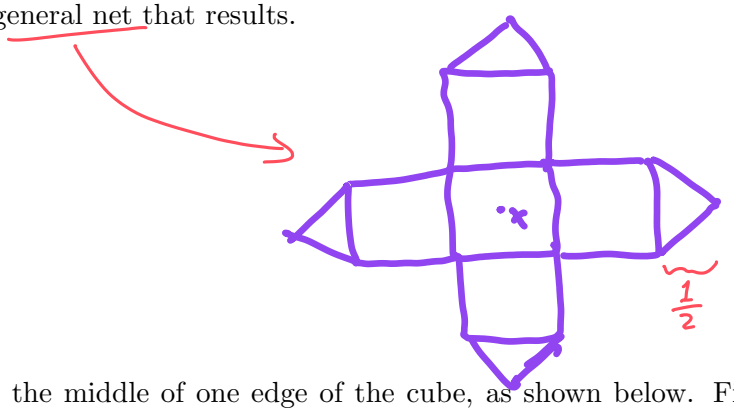
NOTE: similar to the medial axis.

SOURCE UNFOLDING: general net that results when you cut along the cut locus

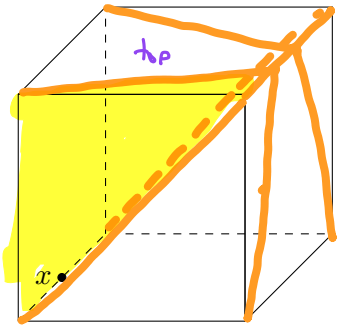
3. (a) Place a point x in the middle of one face of the cube, as shown below. Find the set of points on the surface of the cube that have *more* than one shortest path to x.

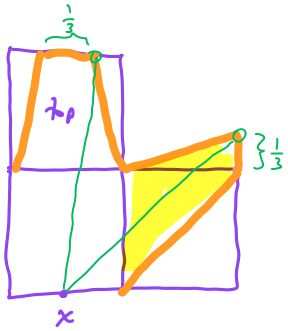


(b) If you cut the cube along the set of points you identified in part (a), can you unfold the cube? Sketch the general net that results.



4. (a) Place a point x in the middle of one edge of the cube, as shown below. Find the set of points on the surface of the cube that have *more* than one shortest path to x.





(b) If you cut the cube along the set of points you identified in part (a), can you unfold the cube? Sketch the general net that results.