Homework 7

MATH 261 Computational Geometry
due 5:00pm on Friday, January 24

Solve the following problems from the textbook, and write your solutions clearly and neatly. Make sure to explain your reasoning and provide mathematical details that support your answers. For a few tips on writing solutions, see this helpful guide for mathematical writing.

These exercises are for everyone, regardless of whether or not you are taking this course for CS elective credit.

You may write or type your solutions electronically, or write them on paper and scan/photograph them. Please use a scanning app to produce a single PDF file containing your solutions. Upload your written solutions to the Homework 7 assignment on Moodle.

1. Let P be a set of n points in the plane. Suppose you want to find the two points in P that are closest together. You could do this naively in quadratic time by computing $\binom{n}{2} = \frac{n^2-n}{2}$ distances between points. Explain how you can find the closest pair of points in $O(n \log n)$ time by constructing a Voronoi diagram.

2. Let R and B be two sets of 2D points, colored red and blue, respectively. We want to determine whether there exists a circle that encloses all the red points and excludes all the blue points. If such a circle exists, we want to find one.

Use the paraboloid $z = x^2 + y^2$ to translate this to a problem into a 3D problem that is easier in the sense that it involves finding a planes rather than finding a circle. State the 3D problem clearly. Explain a solution to the 3D problem gives a circle that solves the 2D problem.

Then design an algorithm for solving the 3D problem. A brute-force inefficient algorithm suffices, as long as it really would always work. Describe your algorithm at a high level, giving enough detail so that your algorithm is unambiguous. You don't need to implement it in code (though you can if you want).

3. Let P be the convex polygon with vertices $(0,0), (10,0), (12,2), (8,4), (3,6), (-3,3)$. Use the algorithm described on page 122 to find the medial axis of P, and draw the stages of the construction.

4. Exercise 5.11

5. Exercise 5.12

6. Exercise 5.16

7. Exercise 5.17

8. *Reflection on creativity*: Give one example of a mathematical idea from this class that you found creative, and explain what you find creative about it. For example, you can choose an instance of creativity you experienced in your own problem-solving, an idea from the textbook, or something you witnessed in another person's reasoning.