**RANDOM VARIABLE:** a real-valued function on a sample space

1. State the possible values of each random variable below, and say whether it is discrete or continuous. You might need to make some assumptions; if so, state your assumptions.

(a) $X$ = the sum of the numbers that appear when two standard dice are rolled

$X \in \{2, 3, 4, \ldots, 12\}$ discrete

(b) $N$ = the number of defective circuit boards in a shipment

$N \in \{0, 1, 2, \ldots\}$ or $\{0, 1, 2, \ldots, \max\}$ discrete

(c) $T$ = the high temperature in Northfield on February 29, 2020

$T \in \mathbb{R}$ or $T \in (\min, \max)$ continuous or limited to discrete values by our ability to measure

(d) $Y$ = the annual income of a randomly-selected person in Minnesota

$Y \in \{0, 1, 2, \ldots\}$ cents/dollars, discrete

(e) $L$ = the length of a fish caught in Lake Itasca

$L \in (0, \infty)$ cm or $L \in (0, 1000)$ cm continuous

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**PROBABILITY MASS FUNCTION (pmf):** of a discrete rv $X$ is defined $p(x) = P(X = x)$

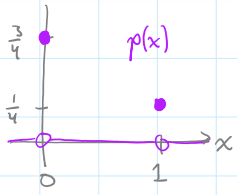
**CUMULATIVE DISTRIBUTION FUNCTION (cdf):** of a discrete rv $X$ with pmf $p(x)$ is defined $F(x) = P(X \le x) = \sum_{y \le x} p(y)$

2. Suppose that one out of every four calls you receive is a robocall. (Assume that all calls are independent.)

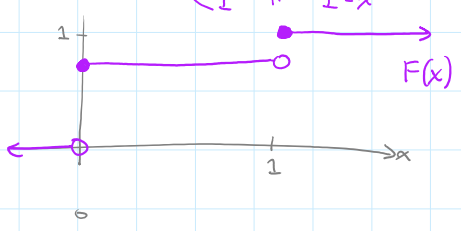
(a) Let $X = 1$ if the next call you receive is a robocall, and let $X = 0$ otherwise. What type of random variable is $X$? State the probability mass function (pmf) and cumulative distribution function (cdf) of $X$. Then sketch each function.

$X \in \{0, 1\}$, so $X$ is a Bernoulli rv

pmf: $p(0) = \frac{3}{4}$, $p(1) = \frac{1}{4}$, $p(x) = 0$ otherwise




cdf: $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{3}{4} & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x \end{cases}$




(b) Let $Y$ be the number of robocalls in the next four phone calls. State the pmf and cdf of $Y$, and sketch

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pmf:

$p(0) = \left(\frac{3}{4}\right)^4 = 0.31$

$p(1) = 4\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 = 0.42$

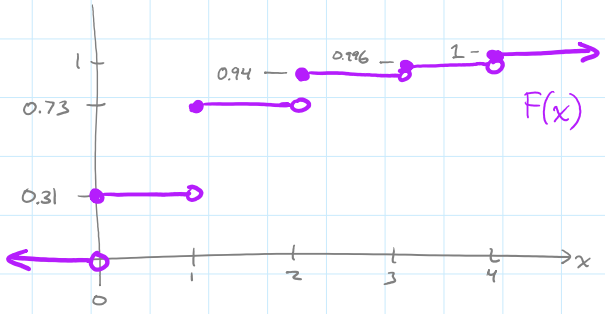
$p(2) = \binom{4}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2 = 0.21$

$p(3) = 4\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right) = 0.05$

$p(4) = \left(\frac{1}{4}\right)^4 = 0.004$

$p(x) = 0$ otherwise

cdf:




3. The cdf for a random variable $X$ is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2 & 0 \le x < 1 \\ 0.5 & 1 \le x < 2 \\ 0.8 & 2 \le x < 4 \\ 1 & 4 \le x \end{cases}$$

(a) What is $P(X = 2)$?

largest possible value of X that is strictly less than 2

$P(X = 2) = F(2) - F(2-) = 0.8 - 0.5 = 0.3$

$P(X \le 2) - P(X < 2)$

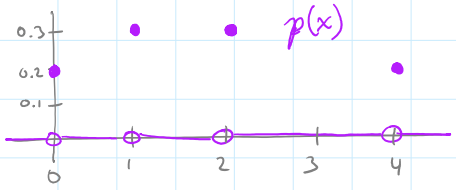
(b) What is $P(X = 3)$?

$P(X = 3) = F(3) - F(3-) = 0.8 - 0.8 = 0$

(c) What is $P(2.5 \le X)$?

$P(2.5 \le X) = 1 - F(2.5-) = 1 - 0.8 = 0.2 = P(X = 4)$

(d) Sketch the pmf of X.




$p(0) = 0.2$
$p(1) = 0.3$
$p(2) = 0.3$
$p(4) = 0.2$

4. Which of the following functions is the pmf for some random variable $X$?

(a) $p(x) = \frac{x^2}{50}$ for $x = 1, 2, \ldots, 5$

$\sum_{x=1}^{5} \frac{x^2}{50} = \frac{1}{50} + \frac{4}{50} + \frac{9}{50} + \frac{16}{50} + \frac{25}{50} = \frac{55}{50} \neq 1$, so NO!

(b) $p(x) = \log_{10}\left(\frac{x+1}{x}\right)$ for $x = 1, 2, \ldots, 9$

YES!

$p(x) \ge 0$

$\sum_{x=1}^{9} p(x) = \sum_{x=1}^{9} \log_{10}\left(\frac{x+1}{x}\right) = \log_{10}\left(\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \frac{10}{9}\right) = \log_{10}(10) = 1$

Benford's Law

5. Which of the following properties must hold for any cdf F(x)? For each property, either say why it must hold or give a counterexample to show that it might not hold.

(a) $\lim_{b\to-\infty} F(b) = 0$ Yes: $P(X \le b)$ must go to zero as $b$ decreases towards $-\infty$.

(b) $\lim_{b\to\infty} F(b) = 1$ Yes: $P(X \le b)$ must go to 1 as $b$ increases towards $\infty$.

(c) $F(x)$ is continuous No: see counterexamples in #2 and #3 above.

(d) $F(x)$ is nondecreasing; that is, if $a < b$, then $F(a) \le F(b)$

Yes: if $a<b$, then $F(a) = P(X \le a) \le P(X \le a) + \underbrace{P(a < X \le b)}_{\text{this is non-negative}} = P(X \le b) = F(b)$

(e) $F(b) = 0.5$ for some value $b$

No: counterexamples in #2 above.