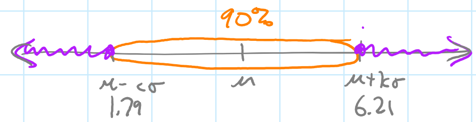
3. The number of equipment breakdowns in a manufacturing plant averages 4 per week, with standard deviation 0.7 per week.

(a) Find an interval that includes at least 90% of the weekly figures for the number of breakdowns.

Chebyshev: $P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$



Choose k such that $\frac{1}{k^2} = 0.1$. So $k^2 = 10$, $k=\sqrt{10}$

$\mu - k\sigma = 4-\sqrt{10}(0.7) = 1.79$

$\mu + k\sigma = 4+\sqrt{10}(0.7) = 6.21$

Then: $P(|X-4| \geq \sqrt{10}(0.7)) \leq \frac{1}{10}$

$P(X \leq 1.79 \text{ or } X \geq 6.21) \leq \frac{1}{10}$

Complement:

$P(1.79 < X < 6.21) \geq \frac{9}{10}$

Interval: $(1.79, 6.21)$ or $[2, 6]$ or $\{2,3,4,5,6\}$

(b) A plant supervisor promises that the number of breakdowns will rarely exceed 7 in a one-week period. Is the supervisor justified in making this claim? Why? YES.

From part (a), we know that in at least 90% of the weeks, there are 6 or less breakdowns.

BINOMIAL RANDOM VARIABLES

$X \sim \text{Bin}(n,p)$ means that X is a binomial random variable that counts the number of successes in a binomial experiment with n trials and success probability p.

pmf: $b(x; n, p) = P(X = x)$ if $X \sim \text{Bin}(n,p)$

cdf: $B(x; n, p) = P(X \leq x)$ " " "

EXAMPLE: X is the number of 1s in ten rolls of standard, fair dice

- $X \sim \text{Bin}(10, \frac{1}{6})$
- $b(x; 10, \frac{1}{6}) = \binom{10}{x} \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{10-x}$ _ _ _ _ _ _ _ _ _ _

ways to arrange x 1's and $10-x$ non-1's

prob. of x 1s

prob. of $10-x$ non-1s

In general: $b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$

In general: $b(x;n,p) = \binom{n}{x} p^x (1-p)^{n-x}$

Is this true? $\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$?

$1 = 1^n = \underbrace{(p+(1-p))^n = p^n + \binom{n}{1} p^{n-1}(1-p) + \binom{n}{2} p^{n-2}(1-p)^2 + \cdots + (1-p)^n}_{\text{binomial expansion or binomial theorem}} = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

- Mean (expectation): $E(X) = np$ die rolls: $E(X) = \frac{10}{6} = 10\left(\frac{1}{6}\right) = np$
- Variance: $E(X) = np(1-p) = npq$ $q = 1-p$

1. Suppose that one of every eight calls you receive is a robocall. Assume that the likelihood of a robocall is independent from one call to the next.

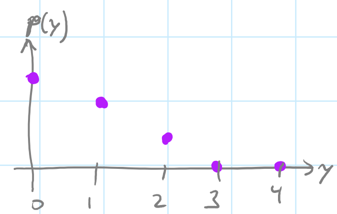
(a) Let $X = 1$ if the next call you receive is from a robocall, and $X = 0$ otherwise. What type of random variable is X? What are its mean and standard deviation?

X is Bernoulli with $p = \frac{1}{8}$ or $X \sim Bin\left(1, \frac{1}{8}\right)$ (Binomial with 1 trial)

$E(X) = \frac{1}{8}$ $Var(X) = \frac{1}{8} \cdot \frac{7}{8} = \frac{7}{64}$ $\sigma_X = \sqrt{\frac{7}{64}} = \frac{\sqrt{7}}{8}$

(b) Let Y be the number of robocalls in the next four phone calls. What type of random variable is Y? Sketch the pmf of Y.

$Y \sim Bin\left(4, \frac{1}{8}\right)$ pmf: $p(y) = b\left(y; 4, \frac{1}{8}\right) = \binom{4}{y}\left(\frac{1}{8}\right)^y \left(\frac{7}{8}\right)^{4-y}$



(c) What are the mean and standard deviation of Y?

$E(Y) = 4\left(\frac{1}{8}\right) = \frac{1}{2}$ $Var(Y) = 4\left(\frac{1}{8}\right)\left(\frac{7}{8}\right) = \frac{7}{16}$

$\sigma_Y = \sqrt{Var(Y)} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$

(d) Suppose that you lose 30 seconds of your time every time you answer a robocall. What is the expected value and standard deviation of the amount of time you will lose when answering the next four phone calls?

$T = 30Y$ (time lost) then $E(T) = \underbrace{E(30Y) = 30E(Y)}_{\text{linearity of expectation}} = 30\left(\frac{1}{2}\right) = 15$ seconds

$Var(T) = Var(30Y) = 30^2 Var(Y) = 30^2 \frac{7}{16} = 393.75$

BINOMIAL CALCULATION

R: dbinom(x,n,p) gives $P(X=x)$ when $X \sim Bin(n,p)$ (pmf)

pbinom(x,n,p) gives $P(X \le x)$ " " " (cdf)

2. Among persons donating blood to a clinic, 85% have Rh^+ blood. Six people donate blood at the clinic on a particular day.

(a) Find the probability that at most three of the six have Rh^+ blood.

to be continued...

(b) Find the probability that at most one of the six does not have Rh^+ blood.

(c) What is the probability that the number of Rh^+ donors lies within two standard deviations of the mean number?

(d) The clinic needs six Rh^+ donors on a certain day. How many people must donate blood to have the probability of obtaining blood from at least six Rh^+ donors over 0.95?