2. Among persons donating blood to a clinic, 85% have $Rh^+$ blood. Six people donate blood at the clinic on a particular day.

(a) Find the probability that at most three of the six have $Rh^+$ blood.

$X \sim \text{Bin}(6, 0.85)$ $\quad P(X \le 3) = \sum_{x=0}^{3} \binom{6}{x} (0.85)^x (0.15)^{6-x} = 0.047$

R: pbinom(3, 6, 0.85)

(b) Find the probability that at most one of the six does not have $Rh^+$ blood.

$P(X \ge 5) = \sum_{x=5}^{6} \binom{6}{x} (0.85)^x (0.15)^{6-x} = 0.776$

1 − pbinom(4, 6, 0.85) = pbinom(1, 6, 0.15)

(c) What is the probability that the number of $Rh^+$ donors lies within two standard deviations of the mean number?

Chebyshev: $P(|X - \mu| \ge k\sigma) \le \frac{1}{k^2}$

Here, $\mu = 6(0.85) = 5.1$, $\sigma = 0.874$

let $k=2$: $P(|X - 5.1| \ge \underbrace{2(0.874)}_{1.75}) \le \frac{1}{4}$


$P(3.35 < X < 6.85) \ge \frac{3}{4}$

$P(X \in \{4,5,6\}) \ge \frac{3}{4}$

We know the pmf of $X$, so:

$P(X \in \{4,5,6\}) = \sum_{x=4}^{6} \binom{6}{x} (0.85)^x (0.15)^{6-x} = 0.953$

pbinom(6, 6, 0.85) − pbinom(3, 6, 0.85)

(d) The clinic needs six $Rh^+$ donors on a certain day. How many people must donate blood to have the probability of obtaining blood from at least six $Rh^+$ donors over 0.95?

Let $Y_n \sim \text{Bin}(n, 0.85)$. We want $n$ such that $P(Y_n \ge 6) \ge 0.95$.

1 − pbinom(5, n, 0.85) $\longrightarrow$ $1 - P(Y_n \le 5) \ge 0.95$

$\sum_{y=6}^{n} \binom{n}{y} (0.85)^y (0.15)^{n-y} \ge 0.95$

Test some n:

$n=8$: $P(Y_8 \ge 6) = 0.895$

$n=9$: $P(Y_9 \ge 6) = 0.966$ Thus, $n=9$ will work.

# POISSON DISTRIBUTION

Suppose we have a sequence of "events" that:

- occur independently
- the average number of events in some fixed time interval is $\mu$ (known).

**EXAMLES:** arrival of emails, phone calls, customers entering a store, emission of radioactive particles, cars passing a spot on a road

POISSON PROCESS

If $X$ is the <u>count</u> of such events occurring in a fixed time interval, then $X$ has a Poisson distribution. Then:

$$P(X=x) = p(x;\mu) = e^{-\mu}\frac{\mu^x}{x!} \quad \text{for } x=0,1,2,3,\ldots$$

check: $$\sum_{k=0}^{\infty} e^{-\mu}\frac{\mu^k}{k!} = e^{-\mu}\underbrace{\sum_{k=0}^{\infty}\frac{\mu^k}{k!}} = e^{-\mu}\left(e^{\mu}\right) = 1$$

Taylor Series for $e^x$:
$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!}$$

$$E(X) = Var(X) = \mu$$

**In R:**

dpois$(x,\mu)$ $\leftarrow$ gives $P(X=x)$

ppois$(x,\mu)$ $\leftarrow$ gives $P(X\le x)$