Negative binomial with $r=1$, $p=3/8$
or Geometric with $p=3/8$

1. Suppose that three out of every eight calls you receive is a robocall, and successive calls are independent. Let X be the number of calls you receive, until (and including) the next robocall.

(a) What is $P(X = 3)$?

$$P(X=3) = \underbrace{\left(\tfrac{5}{8}\right)^2}_{\text{not robocalls}} \underbrace{\left(\tfrac{3}{8}\right)}_{\text{robocall}} = \frac{75}{512} \approx 0.146$$

(b) If n is any positive integer, what is $P(X = n)$?

$$P(X=n) = \underbrace{\left(\tfrac{5}{8}\right)^{n-1}}_{\text{not robocalls}} \underbrace{\left(\tfrac{3}{8}\right)}_{\text{robocall}} = \frac{5^{n-1}\cdot 3}{8^n}$$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}\cdot\frac{5}{8}$

$P(X=3) = \frac{3}{8}\cdot\left(\frac{5}{8}\right)^2$

$P(X=4) = \frac{3}{8}\cdot\left(\frac{5}{8}\right)^3$

$\vdots$

GEOMETRIC SEQUENCE

(c) What is $E(X)$?

value probability

$$E(X) = \sum_{n=1}^{\infty} n \cdot p(n) = \sum_{n=1}^{\infty} n \left(\tfrac{5}{8}\right)^{n-1}\left(\tfrac{3}{8}\right) = \frac{\frac{3}{8}}{\left(1-\frac{5}{8}\right)^2} = \frac{\frac{3}{8}}{\left(\frac{3}{8}\right)^2} = \frac{8}{3}$$

$= \frac{r}{p}$

geometric series: $\displaystyle\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ for $|r|<1$

differentiate: $\displaystyle\sum_{n=1}^{\infty} nar^{n-1} = \frac{a}{(1-r)^2}$ for $|r|<1$

$a = \frac{3}{8}$
$r = \frac{5}{8}$

NEGATIVE BINOMIAL rv with $r=3$, $p=\frac{3}{8}$

2. Let Y be the number of calls until (and including) the third robocall. What is $P(Y = n)$?

$$P(Y=n) = \binom{n-1}{2}\left(\tfrac{3}{8}\right)^2\left(\tfrac{5}{8}\right)^{n-3}\left(\tfrac{3}{8}\right) = \binom{n-1}{2}\left(\tfrac{3}{8}\right)^3\left(\tfrac{5}{8}\right)^{n-3}$$

number of ways to get exactly 2 robocalls in the first n-1 calls

prob. of 2 robocalls

prob. of n-3 non-robocalls

prob. that the n^{th} call is a robocall

	GEOMETRIC	NEG. BINOMIAL
parameters	p	r, p
scenario	number of trials to achieve the first success	number of trials to achieve the r^{th} success
pmf	$(1-p)^{n-1}p$	$\binom{n-1}{r-1}(1-p)^{n-r}p^r$
Mean	$\frac{1}{p}$	$\frac{r}{p}$
Variance	$\frac{1-p}{p^2}$	$\frac{r(1-p)}{p^2}$

INTUITIVELY: $\frac{1}{p}$ trials for each success, on average

CAUTION: Some authors define these variables by the number of failures before the r^{th} success, not the total number of trials.

3. An interviewer must find 10 people who agree to be interviewed. Suppose that when asked, a person agrees to be interviewed with probability 0.4.

(a) What is the probability that the interviewer will have to ask exactly 20 people?

Let $X \sim NB(r=10,\ p=0.4)$. Then $P(X=20) = \binom{20-1}{10-1}(0.4)^{10}(0.6)^{10} = 0.0586$

(b) What are the mean and standard deviation of the number of people that the interviewer will have to ask?

$E(X) = \frac{r}{p} = \frac{10}{0.4} = 25$

$Var(X) = \frac{r(1-p)}{p^2} = \frac{10(0.6)}{(0.4)^2} = 37.5$

$\sigma_X = \sqrt{37.5} = 6.12$

4. If X has a geometric distribution with parameter p, and k is a positive integer, find a simple expression for $P(X > k)$.

$X > k$ means that the first k trials are all failures

Prob. of this is $(1-p)^k$.

Thus: $\boxed{P(X>k) = (1-p)^k}$

also: $P(X>k) = \sum_{n=k+1}^{\infty}(1-p)^{n-1}p$

GEOMETRIC TAIL PROBABILITY

GEOMETRIC TAIL PROBABILITY

5. Robocalls, again. $p=\frac{3}{8}$

(a) What is the probability that none of the first four calls are robocalls?

$$P(X>4) = (1-p)^4 = \left(\tfrac{5}{8}\right)^4 \approx 0.1526$$

(b) If none of the first four calls are robocalls, what is the probability that none of the first seven calls are robocalls?

$$P(X>7 \mid X>4) \underset{\text{def. of conditional probability}}{=} \frac{P(X>7 \text{ and } X>4)}{P(X>4)} = \frac{P(X>7)}{P(X>4)} = \frac{(1-p)^7}{(1-p)^4} = \boxed{(1-p)^3}$$

$$= \left(\tfrac{5}{8}\right)^3 = 0.244$$

$P(X>3)$

OBSERVATION: If $X \sim Geo(p)$ and integers $0<s<t$,

$$P(X>t \mid X>s) = P(X>t-s)$$

MEMORYLESS PROPERTY of a geometric r.v.