# Math 262

1. Let $X_1$ and $X_2$ have joint density $f(x_1, x_2) = 3x_1$, for $0 \le x_2 \le x_1 \le 1$. Let $Y = X_1 - X_2$. Use the following steps to find the density of $Y$.

   (a) Identify the possible values of $Y$.

   (b) Sketch the graph $Y = y$ in the $x_1 x_2$-plane.

   (c) Find the region $R$ in the $x_1 x_2$-plane where $Y \le y$.

   (d) Find the cdf $F_Y(y)$ by integrating the joint density of $X_1$ and $X_2$ over the region $R$.

   (e) Differentiate $F_Y(y)$ to obtain the density $f_Y(y)$.

2. Let $X_1$ and $X_2$ be uniformly distributed on the region of the $x_1x_2$-plane defined by $0 \le x_1$, $0 \le x_2$, and $x_1 + x_2 \le 1$. Let $Y = X_1 + X_2$. Find the density of $Y$.

3. The joint density of $X_1$ and $X_2$ is $f(x_1, x_2) = 4e^{-2(x_1+x_2)}$. Find the density of $Y = \frac{X_1}{X_1+X_2}$.

4. Let the point $(X, Y)$ be randomly selected in the first quadrant of the $xy$-plane according to the density $f(x, y) = \frac{4}{\pi}e^{-x^2-y^2}$. Let $R$ be the distance from $(X, Y)$ to the origin. Find the density of $R$.