

2. The joint pdf of X and Y is $f(x, y) = 3x$, for $0 \leq y \leq x \leq 1$.

(a) What is the conditional distribution of X given $Y = y$?

(b) What is $E(X | Y = y)$?

(c) What is $\text{Var}(X | Y = y)$?

3. For continuous random variables X and Y , show that $E(E(X | Y)) = E(X)$.

4. The number of eggs N found in nests of a certain species of turtles has a Poisson distribution with mean λ . Each egg has probability p of being viable, and this event is independent from egg to egg. Find the mean and the variance of the number of viable eggs per nest.

5. Simulate 10,000 averages, each of k samples from a $\text{Unif}[0, 1]$ distribution. Make a histogram of the 10,000 averages. Start with $k = 1$ and then try larger values of k . How does the shape of the histogram depend on k ?

Here is some code for making such a histogram in *Mathematica*:

```
averages = Table[ Mean[ RandomVariate[ UniformDistribution[], k ] ], 10000 ]  
Histogram[averages]
```

Here is similar code in R:

```
averages <- replicate( 10000, mean(runif(k)) )  
hist(averages)
```

Sketch the shape of your histograms:

- Repeat the previous simulation, but now replace $\text{Unif}[0, 1]$ with a different distribution of your choice. What is the shape of the histogram? How does it depend on k ?

★ **BONUS:** If X and Y are independent binomial random variables with identical parameters n and p , calculate the conditional expected value of X given that $X + Y = m$.